

BEHAVIOR OF WTEE BEAMS IN FLEXURE

by

Laura Elizabeth Volle

B.S. in Civil Engineering, University of Pittsburgh, 2001

Submitted to the Graduate Faculty of

School of Engineering in partial fulfillment

of the requirements for the degree of

Master of Science in Civil Engineering

University of Pittsburgh

2003

UNIVERSITY OF PITTSBURGH

SCHOOL OF ENGINEERING

This thesis was presented

by

Laura Volle

It was defended on

April 23, 2003

and approved by

Dr. Jeen-Shang Lin, Associate Professor, Department of
Civil and Environmental Engineering

Dr. Julie M. Vandenbossche, Assistant Professor, Department of
Civil and Environmental Engineering

Thesis Advisor: Dr. Christopher J. Earls, Associate Professor, Department of
Civil and Environmental Engineering

ABSTRACT

BEHAVIOR OF WTEE BEAMS IN FLEXURE

Laura Volle, M.S.

University of Pittsburgh, 2003

The current US steel building specifications (ASD and LRFD) make the fundamental assumption that constituent cross-sectional plate components of WTEEs in flexure do not buckle as individual elements. Rather, any buckling that occurs during bending is more global in nature. However, new research and experiments (Corona and Ellison 1997, 1997a, 1998) are proving that this is an oversimplification, and refute the current assumptions. The main objective of the current study is to investigate the general response and governing limit states exhibited by WTEE beams at their maximum load. Recommendations are subsequently made for changes to the current US steel building design specifications as they pertain to the flexural response of WTEEs.

ACKNOWLEDGMENTS

I would like to thank my advisor, Dr. Christopher J. Earls, for his patience, guidance, and constant support throughout my undergraduate and graduate career. I am honored to have been able to work under him.

I would also like to thank my committee, Dr. Jeen-Shang Lin and Dr. Julie Vandenbossche.

I would like to thank my husband and best friend, Louis Volle. Without his compassion and encouragement, none of this would have been possible. He has given me strength and belief in myself. I especially want to thank him for his patience with me throughout this endeavor.

To my mother, Christine Andrus, I would like to say thank you for all of the love, encouragement and support ever thought possible.

This work is dedicated to my father, and my guardian angel, Charles Andrus. He taught me that I could do anything that my heart desired.

TABLE OF CONTENTS

1.0	INTRODUCTION	1
1.1	Background and Literature Review	4
1.2	Scope.....	14
1.3	Thesis Organization	15
2.0	CURRENT DESIGN PROVISIONS FOR A WTEE.....	16
2.1	Derivation of WTEE-LTB Equation.....	20
3.0	FINITE ELEMENT ANALYSIS	31
3.1	Nonlinear Finite Element Analysis.....	33
3.1.1	Nonlinear Equilibrium Equations	34
3.1.2	Nonlinear Finite Element Solution Techniques.....	35
3.2	Element Selection	40
4.0	FINITE ELEMENT MODEL	42
4.1	Geometry of Finite Element Model	42
4.2	Finite Element Mesh	45
4.3	Imperfection Seed	46
4.4	Material Property Definitions	47
5.0	PARAMETRIC STUDIES AND RESULTS	49
5.1	Parametric Studies	50
5.2	Presentation of Results.....	52

5.3	Increasing the Thickness of the Flange.....	55
5.4	Span to Depth Ratio	58
6.0	CONCLUSIONS.....	60
	APPENDIX A.....	61
	Input for ABAQUS	61
	Appendix A2: Development Formulas	68
	Appendix A3: Development Spreadsheet.....	75
	Appendix A4: Input Files.....	82
	APPENDIX B	101
	Output From ABAQUS	101
	BIBLIOGRAPHY	126

LIST OF TABLES

Table 1 Results of the First Three Trials	53
Table 2 Parametric Studies with Acceptable Span to Depth Ratio.....	54
Table 3 Increase of t_f by 20%	56
Table 4 Increase of t_f by 40%	57

LIST OF FIGURES

Figure 1 Illustration of WTEE Beam.....	1
Figure 2 Definition of Rotation Capacity	3
Figure 3 Negative Moment Induced Upon WTEE Beam.....	5
Figure 4 Bending Machine as Developed by Corona and Ellison.....	6
Figure 5 Coordinate System Employed by Corona and Ellison	7
Figure 6 Measurement of Rotations.....	8
Figure 7 Discretized Model	9
Figure 8 Typical Moment Curvature Response.....	11
Figure 9 Positive and Negative Bending Tests.....	12
Figure 10 Concentrated Moment Loading on a Thin Walled Open Cross-Section.....	20
Figure 11 WTEE Dimensions.....	29
Figure 12 Schematic of Newton Raphson Solution Method	36
Figure 13 Typical Unstable Static Response	39
Figure 14 Schematic of Riks-Wempner solution Method	40
Figure 15 Order of Numbering for a 9-Node Element	41
Figure 16 WTEE Beam Model	44
Figure 17 Mesh Surface Planes.....	45
Figure 18 True Stress versus True Strain (Logarithmic Strain).....	48
Figure 19 WTEE Beam Dimensions.....	51

Figure 20 Failure Mode of WTEE Beam with a Low Value of L_{pd}/d	59
Figure B1 Moment vs. Rotation for WTEE $b_f/d=0.6$, $t_w=1.0''$	102
Figure B2 Moment vs. Rotation for WTEE $b_f/d=0.8$, $t_w=1.0''$	103
Figure B3 Moment vs. Rotation for WTEE $b_f/d=1.0$, $t_w=1.0''$	104
Figure B4 Moment vs. Rotation for WTEE $b_f/d=1.2$, $t_w=1.0''$	105
Figure B5 Moment vs. Rotation for WTEE $b_f/d=1.4$, $t_w=1.1''$	106
Figure B6 Moment vs. Rotation for WTEE $b_f/d=1.6$, $t_w=1.17''$	107
Figure B7 Moment vs. Rotation for WTEE $b_f/d=1.8$, $t_w=1.22''$	108
Figure B8 Moment vs. Rotation for WTEE $b_f/d=2.0$, $t_w=1.25''$	109
Figure B9 Moment vs. Rotation for WTEE $b_f/d=1.4$, $t_w=1.1''$, $\lambda_p = 1.2(65/\sqrt{F_y})$	110
Figure B10 Moment vs. Rotation for WTEE $b_f/d=1.4$, $t_w=1.1''$, $\lambda_p = 1.4(65/\sqrt{F_y})$	111
Figure B11 Moment vs. Rotation for WTEE $b_f/d=1.6$, $t_w=1.17''$, $\lambda_p = 1.2(65/\sqrt{F_y})$	112
Figure B12 Moment vs. Rotation for WTEE $b_f/d=1.6$, $t_w=1.17''$, $\lambda_p = 1.4(65/\sqrt{F_y})$	113
Figure B13 Moment vs. Rotation for WTEE $b_f/d=1.8$, $t_w=1.22''$, $\lambda_p = 1.2(65/\sqrt{F_y})$	114
Figure B14 Moment vs. Rotation for WTEE $b_f/d=1.8$, $t_w=1.22''$, $\lambda_p = 1.4(65/\sqrt{F_y})$	115
Figure B15 Moment vs. Rotation for WTEE $b_f/d=2.0$, $t_w=1.25''$, $\lambda_p = 1.2(65/\sqrt{F_y})$	116
Figure B16 Moment vs. Rotation for WTEE $b_f/d=2.0$, $t_w=1.25''$, $\lambda_p = 1.4(65/\sqrt{F_y})$	117
Figure B17 Boundary Conditions and Applied Loading	118
Figure B18 Typical Deformed Shape	119

Figure B19 Typical Seed Imperfection.....	120
Figure B20 Von Mises Stresses for WTEE $b_f / d = 0.6$, $t_w = 1.0''$	121
Figure B21 Von Mises Stresses for WTEE $b_f / d = 1.8$, $t_w = 1.22''$	122
Figure B22 Von Mises Stresses for WTEE $b_f / d = 2.0$, $t_w = 1.25''$	123
Figure B23 Von Mises Stresses for WTEE $b_f / d = 1.8$, $t_w = 1.22''$, $\lambda_p = 1.2(65 / \sqrt{F_y})$	124
Figure B24 Von Mises Stresses for WTEE $b_f / d = 1.8$, $t_w = 1.22''$, $\lambda_p = 1.4(65 / \sqrt{F_y})$	125

1.0 INTRODUCTION

WTEE beams are typically made by slicing a hot rolled W shape in half length-wise; thereby producing two individual T shaped steel members known as a WTEE. These sections are most commonly used as chord members in both roof and floor trusses. In this application, the mono-symmetric structural shape is subjected to both axial and flexural loads. It is imperative that the member has the ability to resist the applied bending loads. The bending resistance can be achieved either through having the stem in compression and the flange in tension, or the converse. In the present research, the former case will be considered only.

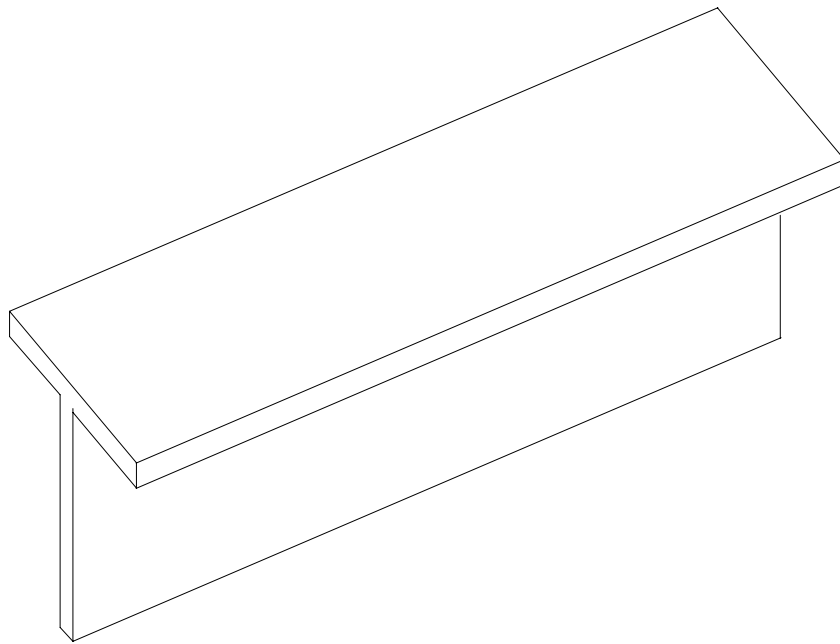


Figure 1 Illustration of WTEE Beam

Buckling during flexure can be classified into three major categories, lateral-torsional buckling, local buckling, or distortional buckling. Lateral-torsional buckling is the simultaneous out-of-plane deflection and twisting of the beam without deformation of the cross section. Long beams generally buckle in a lateral-torsional buckling (LTB) mode. The wavelength of the buckling wave for LTB is the longest of the three types of flexure induced instability.

Local buckling is generally plate buckling in the flange and/or web of the beam without overall lateral deflection or twisting. Any significant deformations are limited to a small region along the length of the beam (i.e. local buckling is a short wave-length manifestation).

Distortional buckling is an intermediate wavelength mode involving characteristics consistent with a combination of the two foregoing buckling modes. Distortional buckling is the simultaneous occurrence of lateral deflections and cross sectional distortion at buckling. Short beams with slender webs are quite susceptible to this buckling mode. The buckling characteristics depend on a variety of geometric and loading parameters.

The current US steel building specifications (ASD and LRFD) make the fundamental assumption regarding the governing response of WTEE beams that plate components of the WTEEs do not buckle as individual elements. Rather, any buckling that occurs due to bending is more global in nature. However, new research and experiments (Corona and Ellison 1997, 1997a, 1998) are proving that this is an oversimplification, and refute the current assumptions.

Rotation capacity is one measure of structural ductility, or deformation capacity, defined by ASCE as $R = \{(\theta_u / \theta_p) - 1\}$ where θ_u is the rotation when the moment capacity drops below M_p on the unloading portion of the $M-\theta$ plot and θ_p is the theoretical rotation at which the full plastic capacity is achieved based on elastic beam stiffness.

This definition is described graphically in Figure 2. In this figure, ϑ_1 corresponds to ϑ_p , and ϑ_2 corresponds to ϑ_u in the ASCE definition.

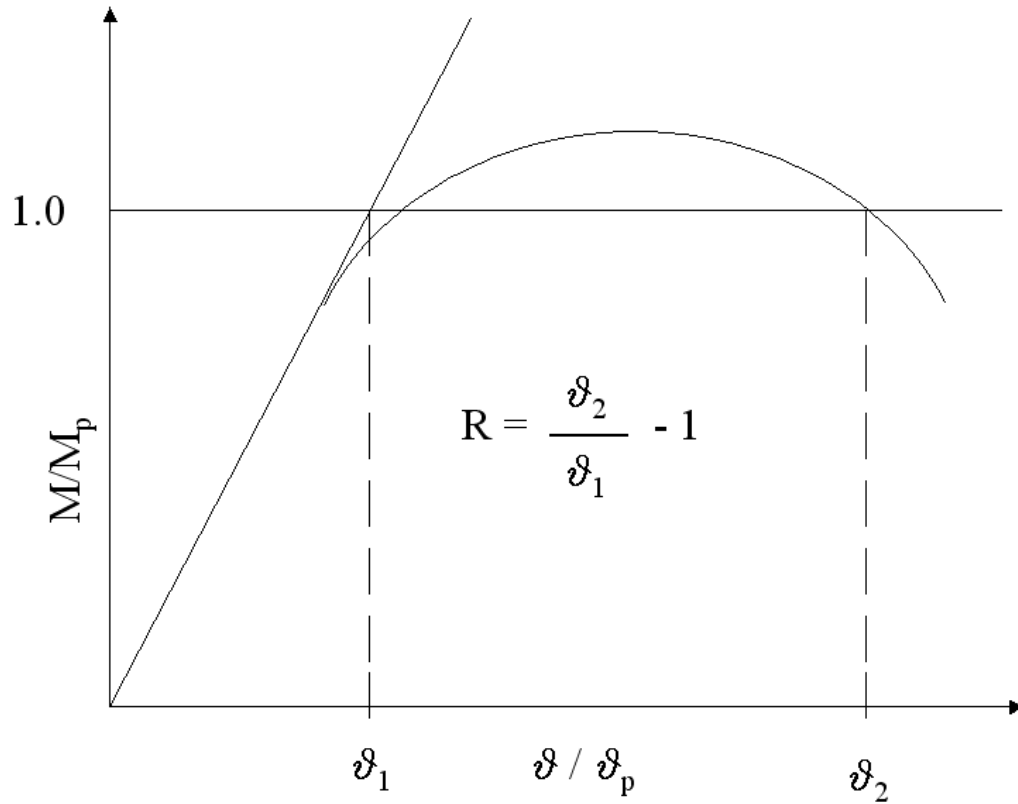


Figure 2 Definition of Rotation Capacity

The main objective of the current study is to investigate the general response and governing limit states exhibited by WTEE beams at their maximum load. Information on cross-sectional rotation capacity will be used to make recommended changes to the current US steel building design specifications.

1.1 Background and Literature Review

WTEE sections are most commonly used as chord members in both roof and floor trusses. In this situation, the mono-symmetric structural shape is subjected to both axial and flexural loads. Detailed, basic research on the response and buckling of such structural members within the plastic range of the material of the steel material response is relatively sparse. The current US steel building specifications (ASD and LRFD) make the fundamental assumption regarding the governing response of WTEE beams that cross-sectional plate components of the WTEEs do not buckle as individual elements. Rather, any buckling that occurs due to bending is more global in nature. However, new research and experiments (Corona and Ellison 1997, 1997a, 1998) are proving that this is an oversimplification, and refute the current assumptions.

Mark S. Ellison and Edmundo Corona (1997) have cooperatively explored the behavior of WTEE beams in flexure. They set out to prove that long beams buckle in a Lateral Torsional Buckling (LTB) mode while short beams buckle locally due to large distortions of the flanges and web. It was also believed that the sign of the bending moment (top of the web in tension or compression) would influence the buckling load and mode. A series of experiments were conducted on the plastic buckling and collapse of WTEE-beams under pure bending. Comparisons between the experimental and numerical results were made to verify results.

The experiment utilized A36 hot-rolled structural steel plates built-up into a WTEE shape. The nominal dimensions of the shape were as listed:

- $b = h = 36mm(1.5")$
- $t_w = t_f = 4.8mm(3/16")$
- $\frac{h}{t} = 8$
- $\frac{L}{h}$ Ranged from 7.5 to 20

Negative moment was induced in the WTEE, thereby putting the stem in compression (see Figure 3).

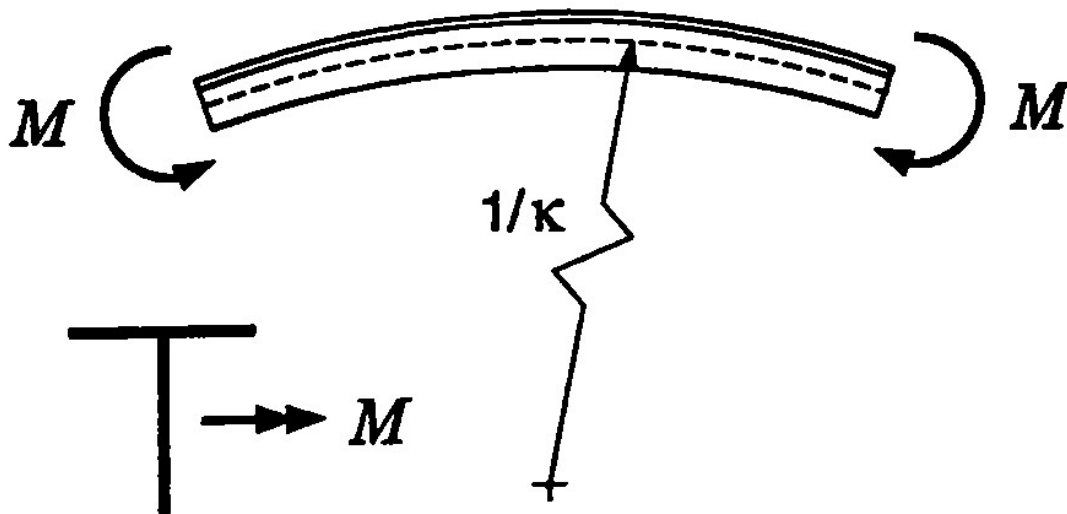


Figure 3 Negative Moment Induced Upon WTEE Beam

A schematic diagram of a two-roller system. A horizontal beam is supported by two rollers, labeled 3 and 10. A curved beam is positioned above the horizontal beam, with its ends resting on the rollers. The curved beam is labeled 9. Two rectangular blocks, labeled 2 and 4, are positioned on the curved beam. Arrows indicate forces or moments: M is applied at the ends of the curved beam, and arrows point from the blocks towards the beam. The diagram is labeled (b).

6

The boundary conditions employed in the experiment allowed no displacements along the y and z-axes (as defined in Figure 5). Additionally, no rotations about the x and z-axes were allowed at the beam-ends. The rotations about the y-axis at the two nodes were equal and were the prescribed loading variables. Translation along the x-axis was prevented at the junction of the flange and the web at the mid-span of the beam.

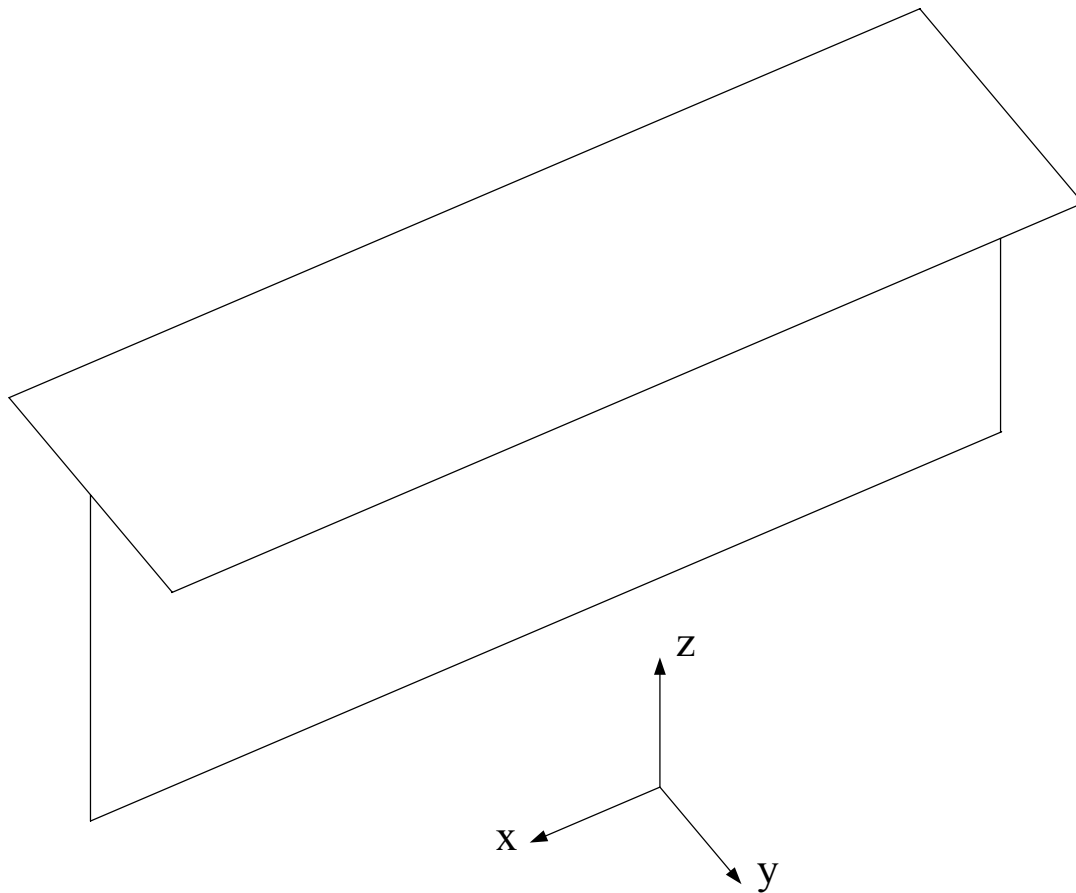


Figure 5 Coordinate System Employed by Corona and Ellison

The setup allowed the specimen to be subjected to pure bending and the direction of bending could be reversed in order to be able to prescribe cyclic bending histories employing full loading reversal. The sum of the rotation of the sprockets was directly proportional to the displacement of the actuators. The device naturally operated in a manor in which the sum of the rotation of the sprockets was the control variable. Rotation of the sprockets was measured independently using rotary variable inductance transducers (RVIT's). The measurements were represented by $k = \frac{\theta_1 + \theta_2}{L}$ where θ_1 and θ_2 are the rotations at the two sprockets. Throughout the experiment, the values of moment, curvature, and torque were recorded every time the curvature changed by 0.004 m^{-1} (0.001 in^{-1}) or the moment changed by $2.3 \text{ N}\cdot\text{m}$ ($20 \text{ lb}\cdot\text{in}$).

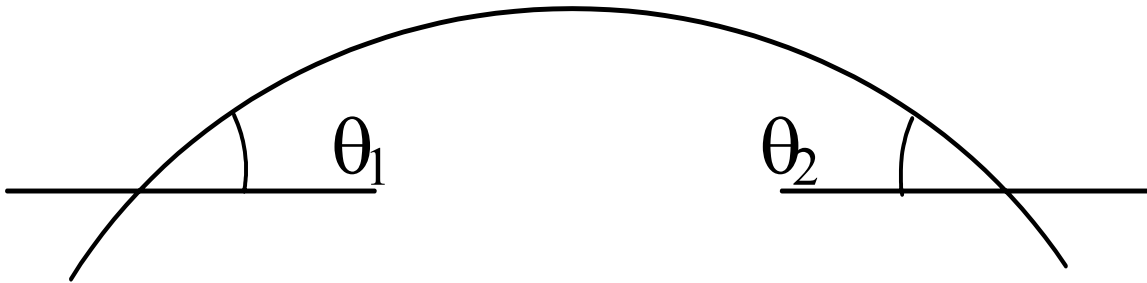


Figure 6 Measurement of Rotations

The results produced by the experiment were compared with numerical results obtained from a finite element model of the experimental specimen. The commercial code ABAQUS was employed to carry out the finite element analysis.

The beams were discretized using S4R nonlinear shell elements. These are shear-flexible, reduced integration, quadrilateral shell elements, which allow for large rotations and finite strains. The two ends of the beam were modeled with rigid elements, R3D4. These elements create rigid bodies with the same cross-sectional geometry as the beam. Rigid body reference nodes were defined at the centroid of the cross section between the two rows of rigid elements at both ends of the mesh. The rigid ends were used to replicate the ends of the beam, which were secured in the experimental apparatus. The boundary conditions applied to the model were identical to those in the experimental model.

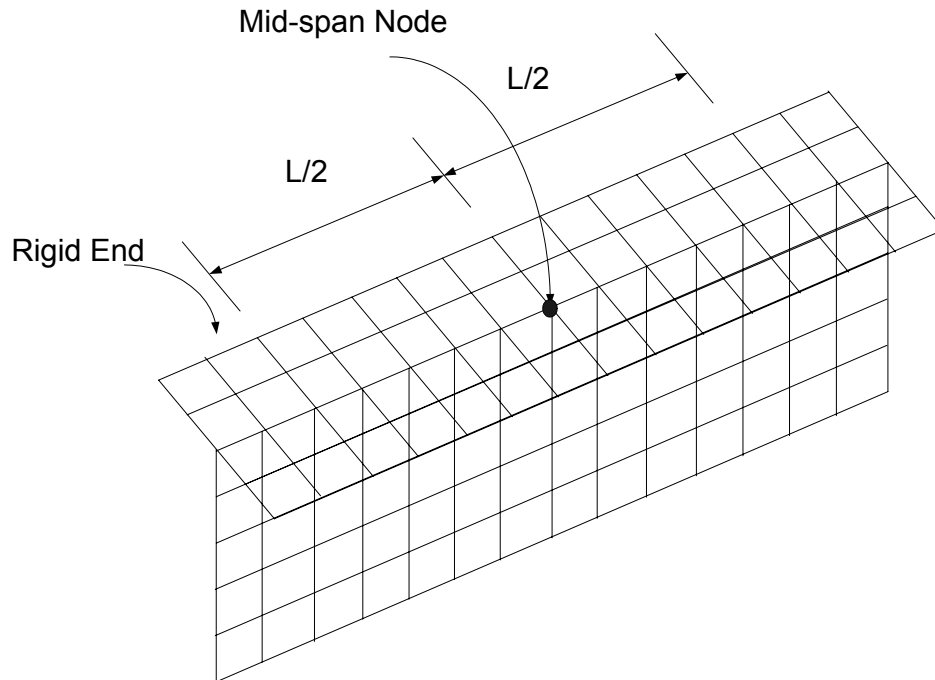


Figure 7 Discretized Model

The physical experiment produced results that indicated that the collapse modes of the beams had components of lateral-torsional buckling and distortion of the web to different degrees. Special attention was paid to the role that the deformation of the cross section played in the response, buckling and collapse of the beams. It was found that the deformation of the cross section depended on the direction of the bending and affected the M-K (moment-curvature) response. If the beam was bent with the tip of the web in tension, significantly higher curvatures were achieved before collapse and no local buckles formed in the curvature range tested. Collapse occurred at relatively low curvatures when the tip of the web was in compression. In this case, a ripple developed in the deflection of the web in all cases considered. This ripple, however, did not have a well-defined wavelength, and was strongly influenced by the initial deflection of the web prior to loading. The ripple could be detected at nearly the same curvature ($K/K_1 \approx 1$) in all specimens despite variations in length. Initially, the amplitude of the ripple grew uniformly with curvature along the specimen at a gradual rate. At higher curvatures the ripple growth accelerated, interacted with the roll, and then localized, which led to a limit-moment instability and the formation of a local buckle. The growth of the roll was accompanied by a rapid increase in torque. The interaction between the ripple and the roll depended in the length of the specimen. (Corona & Ellison, May 1997)

Lateral torsional buckling roll characteristics depended upon the ratio L/h of the specimen. When $L/h = 20$, LTB characteristics were present, however at $L/h=10$, no LTB roll characteristics were present. In these cases, the most severe feature of the ripple pattern initiated the local buckling mechanism. Further bending caused the web deformation and roll to grow

concurrently at an accelerated rate, which eventually formed a local buckle. (Corona & Ellison, May 1997)

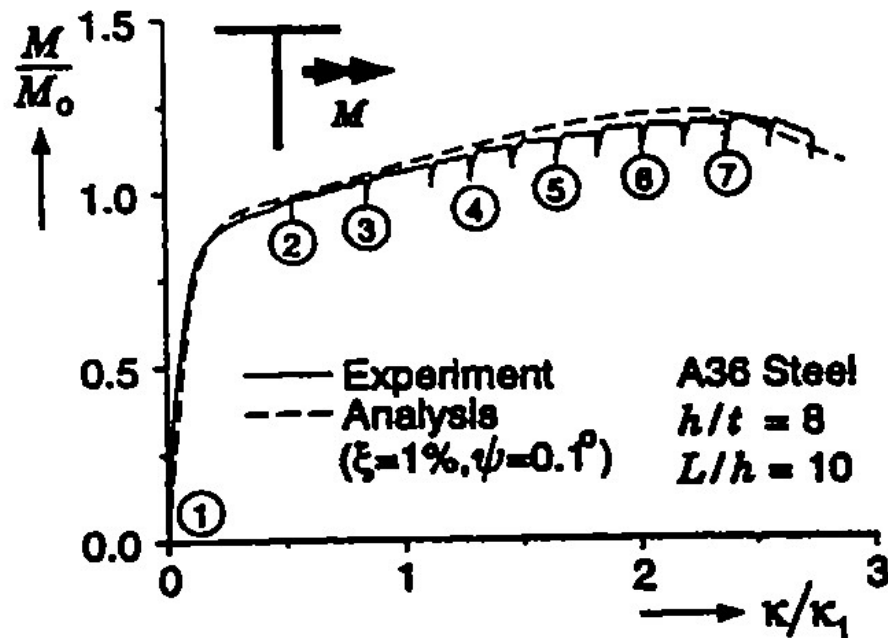


Figure 8 Typical Moment Curvature Response

The predictions of the response of the beams agreed well with experimental measurements. Predictions for both the moment-curvature response and the cross section deformation parameter were consistent with experimental results for moderate curvatures. The finite element bifurcation analysis predicted the curvature at which the ripple appeared, which was in good agreement with experimental observations. (Corona & Ellison, May 1997)

Further experimental data has been gathered on the response of WTEE beams placed in both positive and negative flexure. In particular, Duane S. Ellifritt, Gregory Wine, Thomas Sputo, and Santosh Samuel explored the behavior of WTEE beams placed in both positive and negative flexure.

Specimens with a 7-foot span were loaded using a mechanically driven universal testing machine. The nominal sections used were WT5x13, WT6x13, and WT6x15. A load cell was used to measure loads, and deflections were measured using an LVDT. Five of the eight experiments performed were for positive bending, while the remaining three had the ends fixed creating negative bending at the supports, see figure 9.

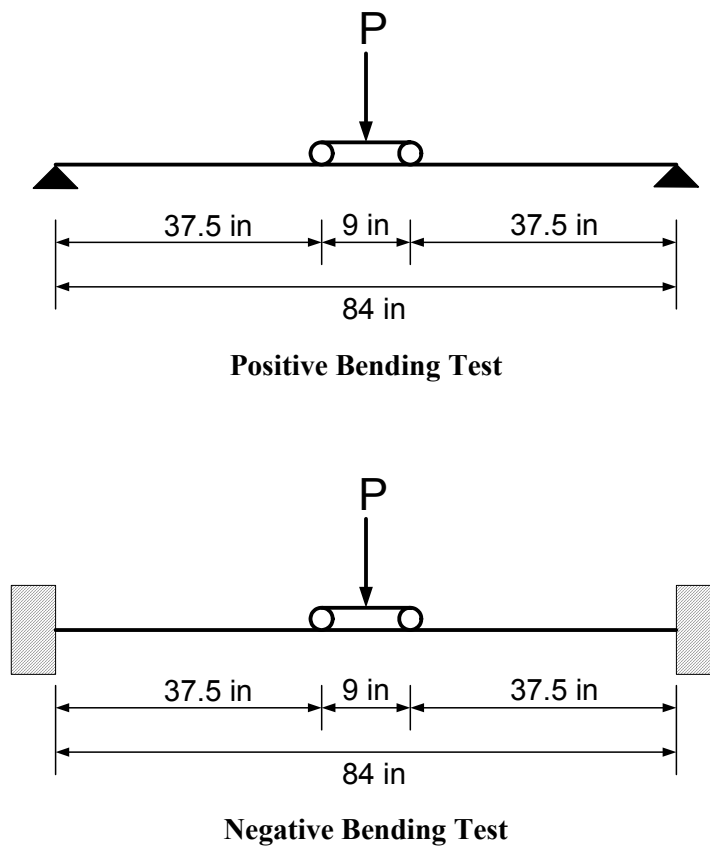


Figure 9 Positive and Negative Bending Tests

Load versus deflection curves and moment versus deflection diagrams were developed from the experimental measurements of the positive bending tests. The deflections followed the predicted linear-elastic deflections quite well until the initiation of yielding in the stem at M_y , then the deflections became nonlinear; beginning to gradually increase until the plastic capacity of the section was reached.

The negative bending test sections failed in the region of negative bending where the stem was in flexural compression. The measured deflections generally followed the predicted elastic deflections once the slippage of the specimen in the supports was considered.

Ellifritt et al. used the results obtained from the experiments conducted to reevaluate the LRFD and ASD specifications. It is noted that the LRFD Equation F1-15 is the limiting lateral buckling equation for WTEE strength in both positive and negative bending. Ellifritt et al. state that in positive bending, it is impossible at practical beam lengths to develop this elastic strength before a plastic hinge is formed. It was observed that a WT6x20 will not experience lateral buckling with the flange in compression until the unbraced length reaches 75 feet. They continue on to observe that it would probably be better just to note the capacity in positive flexure to be $1.5M_y$ and to eliminate the use of Equation F1-15 for positive bending. Additionally, for negative bending, it was observed that for any case where the elastic buckling stress is greater than the yield stress, Equation F1-15 will not control. For cases where the elastic buckling stress is less than the yield stress, lateral buckling will govern for negative bending, and the lesser of M_y and Equation F1-15 should govern. (Ellifritt et al, 1992)

Similar results were recorded for the ASD specification. Ellifritt et al. state that it is unnecessarily conservative to limit the allowable bending stress in the stem for positive bending

to $0.66F_y$. A limit of $0.9M_y$ would be more reasonable and in keeping with the LRFD design criteria; which allow a service load of $0.90F_y$ at a live/dead load ratio of 3/1. Negative bending of WTEEs is not covered in the ASD specification. It was recommended to check both lateral buckling and the strength for specific sections and lengths, for cases where the elastic buckling stress is less than the yield stress. (Ellifritt et al, 1992)

1.2 Scope

The main objective of the current study is to investigate the general response and governing limit states exhibited by WTEE beams at their maximum load. Compactness criteria that ensure attainment of $R=3$ are formulated through the use of experimentally verified nonlinear finite element modeling techniques. Parametric studies are conducted to determine the influence of various geometrical properties on the maximum allowable flexural capacity of WTEEs. The information obtained is used to make recommendations for changes to the current US steel building design specifications.

1.3 Thesis Organization

Section 2 provides an overview of the current provisions used for the design of a typical WTEE beam. Specific equations and limitations will be outlined in this section. The specific finite element modeling methods and techniques will be described in Section 3. Section 3.1 discusses the nonlinear finite element methods used to conduct this research along with an overview of the finite element modeling program ABAQUS. The finite element model that will be analyzed by ABAQUS for this research will be described in Section 4. Section 5 discusses, in detail, the parametric study conducted. The conclusions of this study will be provided in Section 6.

2.0 CURRENT DESIGN PROVISIONS FOR A WTEE

The nominal moment capacity for a WTEE loaded in flexure, such that bending occurs within the plane of the web, is governed by the specification provisions in Chapter F of the American Institute of Steel Construction Load and Resistance Factor Design Manual of Steel Construction (LRFD). This chapter applies to compact and noncompact doubly and singly symmetric prismatic members subject to flexure and shear. Because the current research focuses on flexural design, Section 1 of Chapter F, Design For Flexure, will be explored.

A section is said to be compact if it is capable of developing a fully plastic stress distribution and possesses a rotation capacity of approximately three before the onset of local buckling (AISC LRFD, Second Edition). In order to prevent local or global buckling prior to the attainment of M_p , the beam must be compact and adequately braced (Salmon and Johnson, 1996).

To attain the requisite ductility for compactness, three limit states must be investigated to determine adequacy: lateral-torsional buckling (LTB), local buckling of the compression flange (FLB), and local buckling of the web (WLB). The nominal flexural strength M_n assumes the value is the lowest value obtained from consideration of these three limit states. These limit states depend, respectively, on the beam slenderness ratio L_b/r_y , the width-thickness ratio $b_f/2t_f$ of the compression flange and the width-thickness ratio h/t_w of the web. For convenience, all three measures of slenderness are denoted by λ .

Chapter F, Section 1 states that for unbraced compact beams, noncompact tees and double angles, only the limit states of yielding and lateral-torsional buckling are applicable. It is

noted that the limit states for local buckling are not applicable. Section 1 applies to homogeneous and hybrid shapes with at least one axis of symmetry and which are subject to simple bending about one principal axis. For simple bending, the beam is loaded in a plane parallel to a principal axis that passes through the shear center or the beam is restrained against twisting at load points and supports. Only the limit states of yielding and lateral-torsional buckling are considered in this section. Appendix F1 is applied for lateral-torsional buckling of other singly symmetric shapes and for the limit states of flange local buckling and web local buckling of noncompact or slender-element sections (AISC LRFD Second Edition).

Salmon and Johnson state that a T-section may be thought of as a monosymmetric I-shaped section that has the moment of inertia I_y of one flange equal to zero. Traditionally, both ASD and LRFD have been vague on how a T-section is to be treated. Rolled structural tees will rarely have strength controlled by the lateral-torsional buckling limit state. Whenever a tee section is loaded in the plane of its web (moment about the x-axis) and r_x is less than r_y , there is no limit on laterally unbraced length. Rolled tees are quite often subject to bending about the minor axis. Under this condition, the AISC LRFD-F1 states that the lateral torsional buckling limit state is not applicable (Salmon and Johnson, 1996).

Following this same reasoning, when a WTEE having its flange in compression is bent about its major axis and $r_x > r_y$, Equation F1-4 applies for L_p , the limiting laterally unbraced length for full plastic bending capacity:

$$L_p = \frac{300r_y}{\sqrt{F_{yf}}} \quad (2-1)$$

From LRFD-Appendix Table A-F1.1, when L_b , the laterally unbraced length, does not exceed L_p , and λ , does not exceed the limiting slenderness parameter, λ_p , for the flange local buckling limit state, the yielding limit state controls. It may be expected that the plastic moment, M_p will be reached. However, when the shape factor, $\xi = Z_x / S_x$, is large, too much plastic deformation may occur at service load. The shape factor for WTEEs may be as high as 2.0. As a result of the foregoing, LRFD-F1.2c limits the nominal strength M_n (Salmon and Johnson, 1996):

1. For stem in compression,

$$M_n \leq 1.0M_y \quad (2-2)$$

2. For stem in tension,

$$M_n \leq 1.5M_y \quad (2-3)$$

When L_b exceeds L_r , the elastic lateral-torsional buckling limit state controls. Note that L_r is the value of L_b at which $M_{cr} = M_r$. According to LRFD-F1.2c, Equation F1-15,

$$M_{cr} = \frac{\pi \sqrt{EI_y GJ}}{L_b} \left[B + \sqrt{1 + B^2} \right] \quad (2-4)$$

where

$$B = \pm 2.3 \left(\frac{d}{L_b} \right) \sqrt{\frac{I_y}{J}} \quad (2-5)$$

The plus sign is for the stem in tension and the minus sign is used for the stem in compression: “If the tip of the stem is in compression anywhere along the unbraced length, use the negative value of B ” (LRFD-F1.2c).

LRFD Table B5.1 lists the limiting width-thickness ratios for compression elements. For WTEES the width to thickness ratio is b/t . No limiting width-thickness ratio is given for compact stems of tees. For noncompact stems of tees,

$$\lambda_r = \frac{127}{\sqrt{F_y}} \quad (2-6)$$

For flanges of WTEEs, λ_p is given for compact members, and λ_r is given for noncompact members.

$$\lambda_p = \frac{65}{\sqrt{F_y}} \quad \lambda_r = \frac{141}{\sqrt{F_y - 10}} \quad (2-7a \& b)$$

Values of λ_p for FLB and WLB produce a compact section with rotation capacity of about three, after reaching M_p , before the onset of local buckling, and therefore meet the requirements for plastic analysis of load effects. On the other hand, limiting value of λ for LTB

does not allow for plastic analysis because it does not provide rotation capacity beyond that needed to develop M_p . As a result, the unbraced length in a compact section proportioned with plastic design techniques should not exceed that outlined by LRFD-F1-17:

$$L_{pd} = \frac{\left[3,600 + 2,200 \left(\frac{M_1}{M_2} \right) \right] r_y}{F_y} \quad (2-8)$$

2.1 Derivation of WTEE-LTB Equation

The derivation of the WTEE-LTB equation shown through the use of differential equations for a singly symmetric thin-walled open cross-section subjected to a concentrated moment loading follows from the development presented by Galambos (1972).

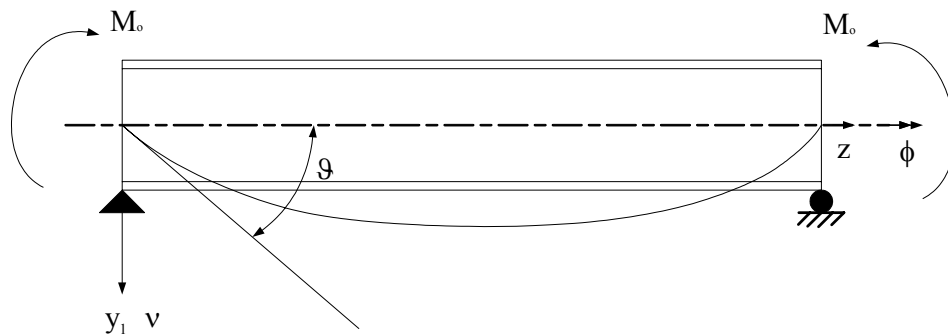


Figure 10 Concentrated Moment Loading on a Thin Walled Open Cross-Section

$$EIu^{IV} + M_x\phi'' + 2M_x'\phi' = 0 \quad (2-9)$$

$$EI_w\phi^{IV} - (GK_T + M_xB_x)\phi'' - M_xu'' = 0 \quad (2-10)$$

Let $M_x = M_o \Rightarrow M_x' = 0$

For a doubly symmetric cross-section,

$$\beta_x = \frac{1}{I_x} \int_{-y_1}^{y_1} \int_{-x_1}^{x_1} y(x^2 + y^2) dx dy = 0 \quad (2-11)$$

hence, the differential equations above will now take the form:

$$EI_yu^{IV} + M_o\phi'' = 0 \quad (2-12)$$

$$EI_w\phi^{IV} - GK_T\phi'' + M_o u'' = 0 \quad (2-13)$$

In the case of the singly symmetric beam, $\beta_x \neq 0$; hence, the differential equations become:

$$EI_yu^{IV} + M_o\phi'' = 0 \quad (2-14)$$

$$EI_w\phi^{IV} - (GK_T + M_o\beta_x)\phi'' + M_o u'' = 0 \quad (2-15)$$

Integrating the first of these two equations twice yields:

$$EI_y u'' + M_o \phi + C_1 z + C_2 = 0 \quad (2-16)$$

Applying simple boundary conditions:

$$u(0) = u(L) = \phi(0) = \phi(L) = u''(0) = u''(L) = \phi''(0) = \phi''(L) = 0 \quad (2-17)$$

at $z = 0$,

$$EI_y u''(0) + M_o \phi(0) + C_1 0 + C_2 = 0 \Rightarrow C_2 = 0 \quad (2-18)$$

at $z = L$,

$$EI_y u''(L) + M_o \phi(L) + C_1 L = 0 \Rightarrow C_1 = 0 \quad (2-19)$$

thus, the first equation can be re-expressed as,

$$u'' = -\frac{M_o \phi}{EI_y} \quad (2-20)$$

Using this result in the second differential equation (2-15) results in,

$$EI_w \phi^{IV} - (GK_T + M_o \beta_x) \phi'' + \frac{M_o^2}{EI_y} \phi = 0 \quad (2-21)$$

This may be re-expressed as:

$$\phi^{IV} - \lambda_1 \phi'' + \phi = 0 \quad (2-22)$$

where,

$$\lambda_1 = \frac{GK_T + M_o \beta_x}{EI_w} \quad \& \quad \lambda_2 = \frac{M_o^2}{E^2 I_y I_w} \quad (2-23 \& 24)$$

The solution of this type of differential equation has the form:

$$\phi = C_1 \cosh \alpha_1 z + C_2 \sinh \alpha_1 z + C_3 \sin \alpha_2 z + C_4 \cos \alpha_2 z \quad (2-25)$$

$C_1, C_2, C_3, \& C_4$ are simply constants of integration while $\alpha_1 \& \alpha_2$ are:

$$\alpha_1 = \sqrt{\frac{\lambda_1 - \sqrt{\lambda_1^2 + 4\lambda_2}}{2}} \quad \alpha_2 = \sqrt{\frac{-\lambda_1 + \sqrt{\lambda_1^2 + 4\lambda_2}}{2}} \quad (2-26 \& 27)$$

Based on the torsional boundary conditions (i.e. no twisting at the ends and no warping restraint at the ends):

$$\phi(0) = \phi(L) = \phi''(0) = \phi''(L) = 0 \quad (2-28)$$

$$\phi = C_1 \cosh \alpha_1 z + C_2 \sinh \alpha_1 z + C_3 \sin \alpha_2 z + C_4 \cos \alpha_2 z \quad (2-29)$$

$$\phi' = C_1 \alpha_1 \sinh \alpha_1 z + C_2 \alpha_1 \cosh \alpha_1 z + C_3 \alpha_2 \cos \alpha_2 z - C_4 \alpha_2 \sin \alpha_2 z \quad (2-30)$$

$$\phi'' = C_1 \alpha_1^2 \cosh \alpha_1 z + C_2 \alpha_1^2 \sinh \alpha_1 z - C_3 \alpha_2^2 \sin \alpha_2 z - C_4 \alpha_2^2 \cos \alpha_2 z \quad (2-31)$$

Applying the boundary conditions:

$$\phi(0) = C_1(1) + C_2(0) + C_3(0) + C_4(1) \quad (2-32)$$

$$\phi''(0) = C_1 \alpha_1^2(1) + C_2 \alpha_1^2(0) + C_3 \alpha_2^2(0) + C_4(-\alpha_2^2) \quad (2-33)$$

$$\phi(L) = C_1 \cosh \alpha_1 L + C_2 \sinh \alpha_1 L + C_3 \sin \alpha_2 L + C_4 \cos \alpha_2 L \quad (2-34)$$

$$\phi''(L) = C_1 \alpha_1^2 \cosh \alpha_1 L + C_2 \alpha_1^2 \sinh \alpha_1 L - C_3 \alpha_2^2 \sin \alpha_2 L - C_4 \alpha_2^2 \cos \alpha_2 L \quad (2-35)$$

These four boundary equations may be expressed in matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ \alpha_1^2 & 0 & 0 & -\alpha_2^2 \\ \cosh \alpha_1 L & \sinh \alpha_1 L & \sin \alpha_2 L & \cos \alpha_2 L \\ \alpha_1^2 \cosh \alpha_1 L & \alpha_1^2 \sinh \alpha_1 L & -\alpha_1^2 \sin \alpha_2 L & -\alpha_2^2 \cos \alpha_2 L \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (2-36)$$

The trivial solution is that $C_1 = C_2 = C_3 = C_4 = 0$. The non-trivial solution requires that the determinant of the coefficient matrix must vanish. Using Cramer's rule, we get (2-37):

$$\begin{vmatrix} 0 & 0 & -\alpha_2^2 \\ \sinh \alpha_1 L & \sin \alpha_2 L & \cos \alpha_2 L \\ \alpha_1^2 \sinh \alpha_1 L & -\alpha_2^2 \sin \alpha_2 L & -\alpha_2^2 \cos \alpha_2 L \end{vmatrix} - \begin{vmatrix} \alpha_1^2 & 0 & 0 \\ \cosh \alpha_1 L & \sinh \alpha_1 L & \sin \alpha_2 L \\ \alpha_1^2 \cosh \alpha_1 L & \alpha_1^2 \sinh \alpha_1 L & -\alpha_2^2 \sin \alpha_2 L \end{vmatrix} = \\ (0) \begin{vmatrix} \sin \alpha_2 L & \cos \alpha_2 L \\ -\alpha_2^2 \sin \alpha_2 L & -\alpha_2^2 \cos \alpha_2 L \end{vmatrix} - (0) \begin{vmatrix} \sinh \alpha_1 L & \cos \alpha_2 L \\ \alpha_1^2 \sinh \alpha_1 L & -\alpha_2^2 \cos \alpha_2 L \end{vmatrix} \\ - \alpha_2^2 \begin{vmatrix} \sinh \alpha_1 L & \sin \alpha_2 L \\ \alpha_1^2 \sinh \alpha_1 L & -\alpha_2^2 \sin \alpha_2 L \end{vmatrix} - \left\{ \alpha_1^2 \begin{vmatrix} \sinh \alpha_1 L & \sin \alpha_2 L \\ \alpha_1^2 \sinh \alpha_1 L & -\alpha_2^2 \sin \alpha_2 L \end{vmatrix} \right. \\ \left. - (0) \begin{vmatrix} \cosh \alpha_1 L & \sin \alpha_2 L \\ \alpha_1^2 \cosh \alpha_1 L & -\alpha_2^2 \sin \alpha_2 L \end{vmatrix} + (0) \begin{vmatrix} \cosh \alpha_1 L & \sinh \alpha_1 L \\ \alpha_1^2 \cosh \alpha_1 L & \alpha_1^2 \sinh \alpha_1 L \end{vmatrix} \right\} = 0$$

Simplifying,

$$\begin{aligned} & -\alpha_2^2 \left[-\sinh \alpha_1 L \alpha_2^2 \sin \alpha_2 L - \alpha_1^2 \sinh \alpha_1 L \sin \alpha_2 L \right] \\ & - \alpha_1^2 \left[-\sinh \alpha_1 L \alpha_2^2 \sin \alpha_2 \sin \alpha_2 L - \alpha_1^2 \sinh \alpha_1 L \sin \alpha_2 L \right] = 0 \end{aligned} \quad (2-38)$$

Expanding and collecting terms,

$$(\alpha_2^2 + \alpha_1^2)^2 \sin \alpha_2 L \sinh \alpha_1 L = 0 \quad (2-39)$$

Because $(\alpha_2^2 + \alpha_1^2)^2 > 0$

$$\sinh \alpha_1 L \sin \alpha_2 L = 0 \quad (2-40)$$

$\sinh \alpha_1 L = 0$ only when $\alpha_1 = 0$, this is the trivial case. Thus, the critical condition is:

$$\sin \alpha_2 L = 0 \quad (2-41)$$

which can only be true when:

$$\alpha_2 L = n\pi \quad (2-42)$$

Thus, for the lowest energy state,

$$\sqrt{\frac{-\lambda_1 + \sqrt{\lambda_1^2 + 4\lambda_2}}{2}} L = \pi \quad (2-43)$$

$$\sqrt{\frac{-GK_T - M_o\beta_x}{EI_w} + \sqrt{\left(\frac{GK_T + M_o\beta_x}{EI_w}\right)^2 + \frac{4M_o^2}{E^2I_yI_w}}}{2}(L) = \pi \quad (2-44)$$

Expanding, we get (2-45):

$$\frac{1}{2} \left[-\frac{GK_T}{EI_w} - \frac{M_o\beta_x}{EI_w} + \left\{ \left(\frac{GK_T}{EI_w} \right)^2 + 2 \frac{GK_T M_o\beta_x}{E^2 I_w^2} + \left(\frac{M_o\beta_x}{EI_w} \right)^2 + \frac{4M_o^2}{E^2 I_y I_w} \right\}^{\frac{1}{2}} \right] L^2 = \pi^2$$

Re-arranging terms:

$$\left(\frac{2\pi^2}{L^2} + \frac{GK_T}{EI_w} + \frac{M_o\beta_x}{EI_w} \right)^2 = \left(\frac{GK_T}{EI_w} \right)^2 + 2 \frac{GK_T M_o\beta_x}{E^2 I_w^2} + \left(\frac{M_o\beta_x}{EI_w} \right)^2 + \frac{4M_o^2}{E^2 I_y I_w} \quad (2-46)$$

After squaring, the left-hand side becomes:

$$\frac{4\pi^4}{L^4} + \left(\frac{GK_T}{EI_w} \right)^2 + \left(\frac{M_o\beta_x}{EI_w} \right)^2 + \frac{4\pi^2 GK_T}{L^2 EI_w} + \frac{4\pi^2 M_o\beta_x}{L^2 EI_w} + 2 \frac{M_o\beta_x GK_T}{E^2 I_w^2} \quad (2-47)$$

Substituting back into the original equation and canceling terms:

$$\frac{4\pi^4}{L^4} + \frac{4\pi^2 GK_T}{L^2 EI_w} + \frac{4\pi^2 M_o\beta_x}{L^2 EI_w} = \frac{4M_o^2}{E^2 I_y I_w} \quad (2-48)$$

This may be re-expressed as:

$$\left(\frac{4}{E^2 I_y I_w}\right) M_o^2 - \left(\frac{4\pi^2 \beta_x}{L^2 EI_w}\right) M_o - \left(\frac{4\pi^2 GK_T}{L^2 EI_w} + \frac{4\pi^4}{L^4}\right) = 0 \quad (2-49)$$

Using the quadratic equation, solve for M_o , the elastic critical moment:

$$M_o = \frac{\pi^2 E \beta_x I_y}{2L^2} \left\{ 1 \pm \sqrt{1 + \frac{4}{\beta_x^2} \left(\frac{L^2 GK_T}{\pi^2 EI_y} + \frac{I_w}{I_y} \right)} \right\} \quad (2-50)$$

For the WTEE shown below:

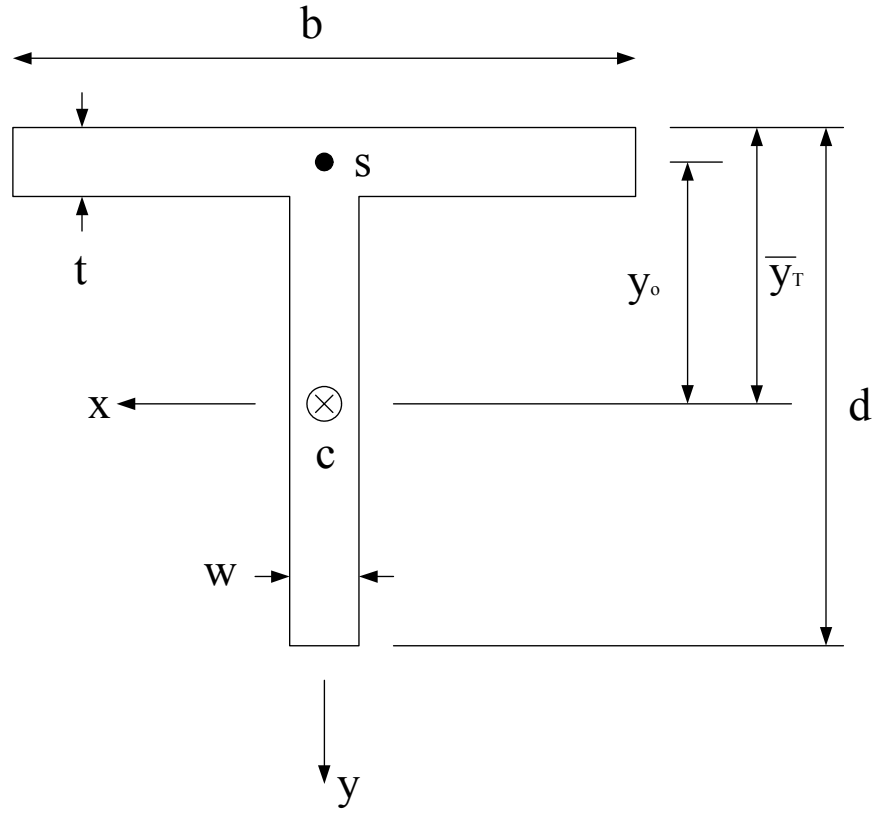


Figure 11 WTEE Dimensions

$$\beta_x = \frac{1}{I_x} \left\{ \frac{w}{4} \left[(d - \bar{y}_T)^4 - (\bar{y}_T - t)^4 \right] - bt \left(\bar{y}_T - \frac{t}{2} \right) \left[\frac{b^2}{12} + \left(\bar{y}_T - \frac{t}{2} \right)^2 \right] \right\} - 2y_o \quad (2-51)$$

Shown below is the elastic buckling comparison between LRFD (F1-15) and the “exact” solution.

Exact Solution:

$$M_o = \frac{\pi^2 E \beta_x I_y}{2L_b^2} \left\{ 1 \pm \sqrt{1 + \frac{4}{\beta_x^2} \left(\frac{L_b^2 GJ}{\pi^2 EI_y} + \frac{C_w}{I_y} \right)} \right\} \quad (2-52)$$

where:

$$\beta_x = \frac{1}{I_x} \left\{ \frac{t_w}{4} \left[(d-y)^4 - (y-t_f)^4 \right] - b_f t_f \left(y - \frac{t_f}{2} \right) \left[\frac{b_f^2}{12} + \left(y - \frac{t_f}{2} \right)^2 \right] \right\} \quad (2-53)$$

AISC LRFD (F1-15):

$$M_{cr} = \frac{\pi \sqrt{EI_y GJ}}{L_b} \left[B + \sqrt{1 + B^2} \right] \quad (2-54)$$

where:

$$B = \pm 2.3 \left(\frac{d}{L_b} \right) \sqrt{\frac{I_y}{J}} \quad (2-55)$$

In both cases above, the positive radical means flange in compression.

3.0 FINITE ELEMENT ANALYSIS

The objective of the current study is to observe the behavior of WTEE beams at ultimate when subjected to bending loads. In order to attain this, it is necessary to determine the cross-section's plastic moment capacity and to observe the ability of the beam to maintain capacity through sufficient member plastic rotations as required for the formation of a collapse mechanism. Success in this regard can be gauged through the use of a load versus deformations plots for the WTEE beams studied. Specifically, a plot of the beam's moment versus rotation response is tracked as shown in Figure 2. A nonlinear displacement based finite element analysis is performed to obtain this plot.

Nonlinear techniques are used because it is expected that the beam will display nonlinear behavior prior to reaching its ultimate capacity. There are two important types of nonlinearities within structural engineering: material and geometric nonlinearities. The first of these relates to the element material properties themselves and how they vary with the deformation of the elements. (i.e. these occur as a result of a nonlinear stress-strain relationship). The second type of a general nonlinearity occurs when the geometric configuration of the assemblage changes sufficiently under load to influence the equilibrium relationships of the structure. This is generally known as geometric nonlinearity.

The commercial multipurpose finite element software package ABAQUS is employed in this research. ABAQUS has the ability to treat both geometric and material nonlinearities that may arise in the model.

The finite element method is powerful when used to solve complex problems. By definition, the method is a generalization of standard structural analysis procedures, which

permits the calculation of stresses and deflections in two- and three-dimensional structures by the same techniques that are applied in the analysis of ordinary framed structures. The finite element method was named based on the fact that, to begin the process of analysis, the structure must be discretized into a finite number of elements, which are interconnected at a finite number of nodal points.

In our case, the WTEE beam is discretized into a system of small bodies, or finite elements. Nodal points, or nodes, then interconnect these elements at points shared by two or more elements. The assembly process results in a system of simultaneous algebraic equations that are solved to obtain the unknown nodal values of the problem. In our case, the unknowns are displacements. These results are then combined to form a solution for the overall member. It is necessary to select elements that are compatible in deformation with adjacent elements. If this property were not satisfied then the elements would deform independently from one another and gaps or overlaps would develop along the edges of the elements; thus causing the idealization to be much more flexible than the actual system.

The final solution for the WTEE beam is generalized as the determination of the displacements at each node, and the stresses within each element, all combined to make up the entire body. The body is therefore discretized into many small elements, and an analysis is performed to arrive at all of the subsequent unknown displacements and stresses.

3.1 Nonlinear Finite Element Analysis

Non-linear analysis is used to iteratively determine the state of equilibrium of the structure subjected to an applied loading. Using this method, the solution cannot be determined from a system of linear equations. Rather, the load is specified as a function of “time”. The “time” is then broken down into intervals that are incrementally applied in small steps so as to trace the non-linear equilibrium response. The values of the accumulated “time” represent Load Proportionality Factors (LPF) related to the evolution of the load history.

In the incremental analysis technique, each step is assumed to be linear with the loading or displacement applied in a series of increments. Each time a new displacement increment is computed, the result is added to the previous displacement of the structural configuration. This configuration is computed for each incremental step. It is these increments in displacement that allow for the observation of changes in the overall model configuration.

The tangent stiffness matrix of the model is used as a means of relating the changes in loads and the changes in displacements in a linearized fashion within an individual load increment (i.e. between two different LPFs). The internal loads and the deformation of the structure at the beginning of the increment are used to compute this stiffness matrix. The tangent stiffness matrix is commonly represented as:

$$[k_T] = [k_o] + [k_p] \quad (3-1)$$

where $[k_o]$ is the conventional linear stiffness matrix for uncoupled bending and axial behavior and $[k_p]$ is the initial stiffness matrix, dependent upon the axial force at the beginning of each load increment.

3.1.1 Nonlinear Equilibrium Equations

The principal of virtual work is the concept whereupon a deformable body in equilibrium is subjected to an arbitrary virtual displacement and reacts with a compatible deformation. It is said that the virtual work of the external forces are equal to the virtual strain energy of the internal stresses. Otherwise stated as:

$$\delta U^{(e)} = \delta V^{(e)} \quad (3-2)$$

where $\delta U^{(e)}$ is the virtual strain energy due to internal stresses, and $\delta V^{(e)}$ is the virtual work of external forces on the element. The external work is due to nodal, surface, and body forces moving through virtual displacements.

The virtual work statement of equilibrium is at the heart of the finite element method in that it is used directly for the formulation for the equilibrium equations used within each increment of the nonlinear analysis.

3.1.2 Nonlinear Finite Element Solution Techniques

In the current study, it is necessary to track the non-linear equilibrium path of the WTEE shape as the load is incrementally applied. The solution to (3-2) must be found using nonlinear finite element solution techniques.

There are several incremental solution techniques, however the two most commonly used are the Newton-Raphson technique, and the Riks-Wempner method. The Newton-Raphson solver is used to approximate a given function in each iteration by a quadratic function, see Figure 12. ABAQUS generally uses Newton's method as a numerical technique for solving the nonlinear equilibrium equations. The motivation for this choice is primarily the convergence rate obtained by using Newton's method compared to the convergence rates exhibited by alternate methods. The number of iterations needed to find a converged solution for a time increment will vary depending on the degree of non-linearity in the system. However, this method is unable to negotiate limit and bifurcation points and hence is not suitable to plot the unloading portion of a nonlinear equilibrium path and is therefore inadequate for this study (Earls, 1995). Alternatively, the Riks-Wempner method is an arc-length method generally used to solve post-buckling problems.

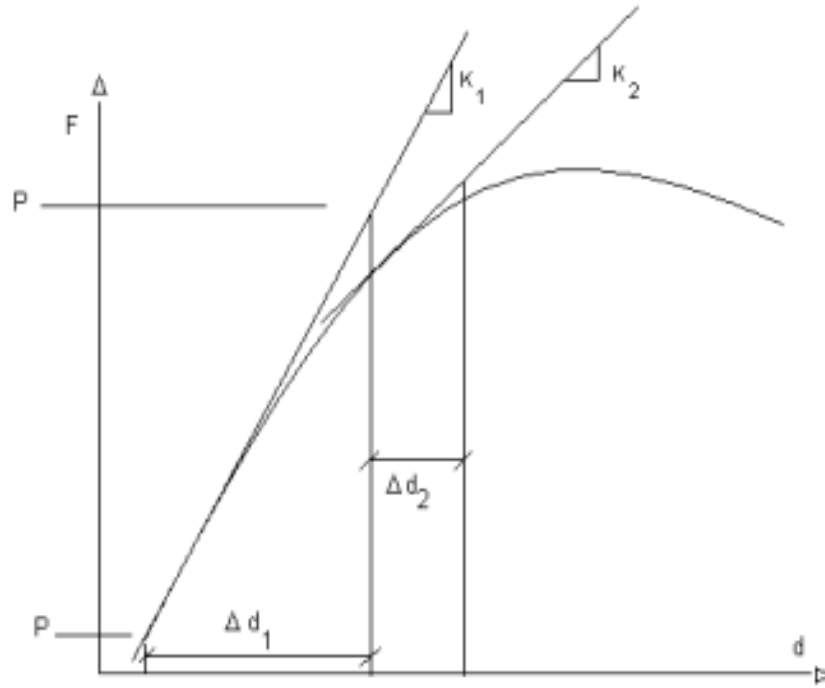


Figure 12 Schematic of Newton Raphson Solution Method

The incremental technique that will be implemented for the current study is the Riks-Wempner method provided by the ABAQUS program. The Riks-Wempner method allows for the equilibrium path to be traced into the unloading portion of the beam response. This method also provides some of the most efficient use of the computational resources during the nonlinear response since step size in an increment is tied to the convergence rate from the previous increment.

3.1.3 Riks-Wempner Method

The Riks method is generally used to predict unstable, geometrically nonlinear collapse of a structure. To analyze a post-buckling problem, it must be turned into a problem with continuous response instead of bifurcation. This effect can be accomplished by introducing an initial imperfection into a “perfect” geometry so that there is some response in the buckling mode before the critical load is reached.

Within ABAQUS there are three ways to define an imperfection. The first is as a linear superposition of buckling eigenmodes. The second is from the displacements of a static analysis. The last method to define an imperfection is by specifying the node number and imperfection values directly on the data lines while creating the input file for ABAQUS.

Unless the precise shape of an imperfection is known, an imperfection consisting of multiple superimposed buckling modes can be introduced. The usual approach involves two analysis runs with the same model definition. The first run performs an eigenvalue buckling analysis on the “perfect” structure to establish probable collapse modes and to verify that the mesh discretizes those modes accurately. The second analysis run will introduce an imperfection in the geometry by adding these buckling modes to the “perfect” geometry by using the IMPERFECTION card in the input file. The lowest buckling modes are assumed to provide the critical imperfections, so usually these are scaled and added to the “perfect” geometry to create the perturbed mesh. The imperfection thus has the form:

$$\Delta x_i = \sum_{i=1}^m \omega_i \phi_i \quad (3-3)$$

where ϕ_i is the i^{th} mode shape and ω_i is the associated scale factor. ABAQUS then performs a geometrically nonlinear load-displacement analysis of the structure containing the imperfection using the Riks-Wempner method.

In simple cases, linear eigenvalue analysis may be sufficient for design evaluation; but if there is concern about material nonlinearity, geometric nonlinearity prior to buckling, or unstable postbuckling response, a load-deflection, or Riks, analysis must be performed to investigate the problem further. (ABAQUS, 1999)

The load magnitude is used as an additional unknown in the Riks method. The loads and displacements are solved for simultaneously. For this reason, another quantity must be used to measure the progress of the solution. ABAQUS uses the arc length along the static equilibrium path in load-displacement space. This approach provides solutions regardless of whether the response is stable or unstable.

As previously mentioned, when the nonlinear static equilibrium solution for unstable problems is desired, ABAQUS uses the Riks-Wempner method. In such cases, ABAQUS allows the effective solution to be determined for situations in which the load and/or the displacement may decrease as the solution evolves. This typical unstable static response is illustrated in Figure 13.

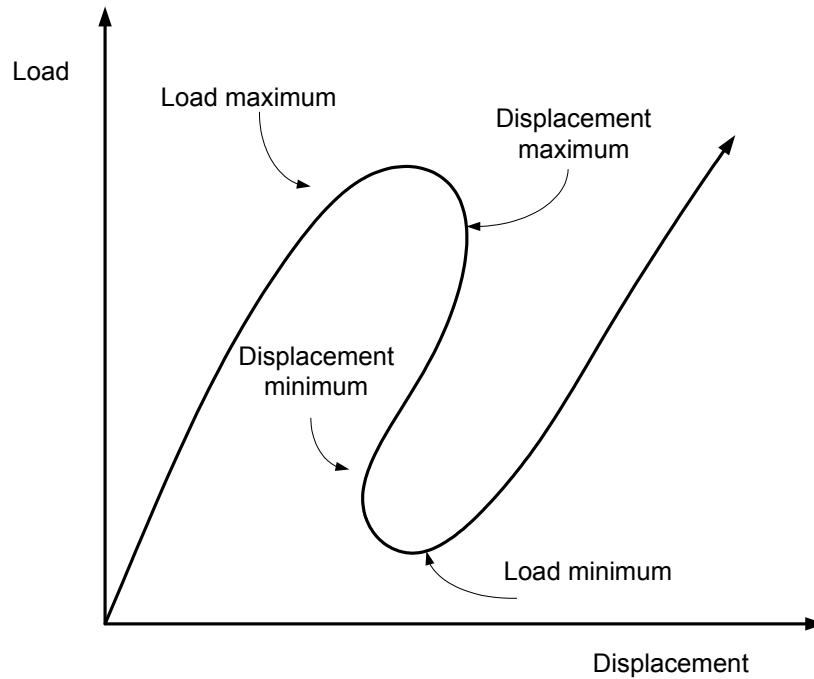


Figure 13 Typical Unstable Static Response

In the modified Riks method, it is assumed that all load magnitudes vary with a single scalar parameter; the loading is proportional to this parameter. The response is additionally assumed to be reasonable smooth and that sudden bifurcations do not occur. The essence of the method is that the solution is viewed as the discovery of a single equilibrium path in a space defined by the nodal variables and the loading parameter. Tracing this path as far as required allows the development of the solution. It is essential to limit the increment size due to the fact that many of the materials, and possibly loadings of interest will have path-dependent response. The increment size for the modified Riks algorithm is limited by moving a given distance along the tangent line to the current solution point and then searching for equilibrium in the plane that passes through the point thus obtained and that is orthogonal to the same tangent line. See Figure 14 for an illustration of the method.

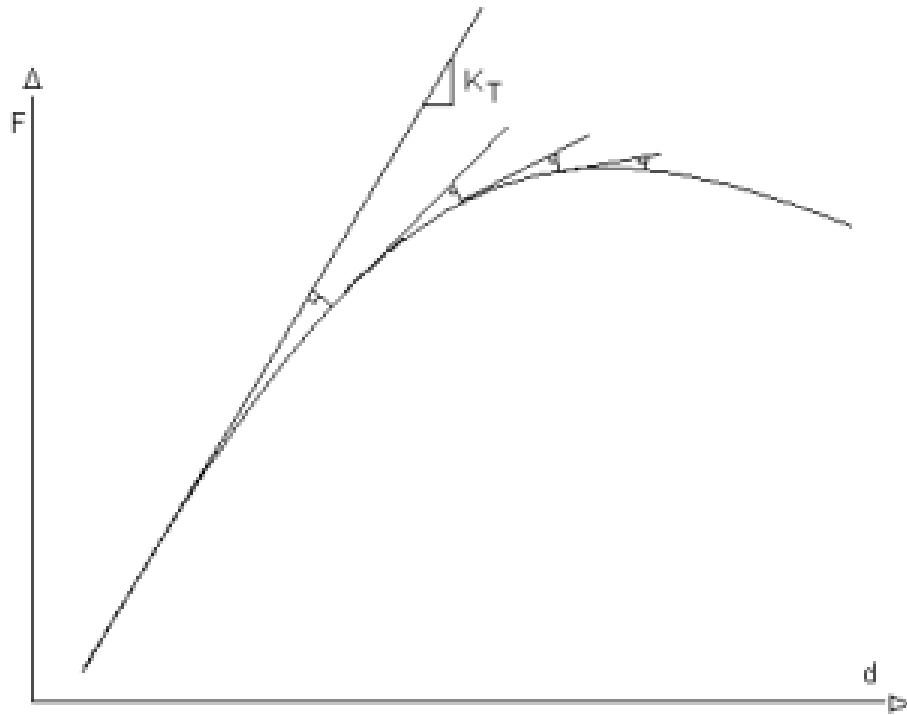


Figure 14 Schematic of Riks-Wempner solution Method

3.2 Element Selection

Shell elements were selected for use in the current study as a result of their ability to model structures in which one dimension, the thickness, is significantly smaller than the other dimensions and the stresses normal to the thickness direction are negligible. The nonlinear shell element used in the current research is the S9R5 (ABAQUS 1999). The naming of the elements

is broken down into four categories. The first letter of the name denotes the element family, the second variable is the number of nodes used to define the element, the third is the type of integration employed in the formulation of the element stiffness matrix, and finally, the last variable in the element name is the number of nodal degrees of freedom. Using this nomenclature, the S9R5 is a shell element consisting of nine nodes using reduced integration, with 5 degrees of freedom per node. The order of numbering the nodes is illustrated in Figure 15.

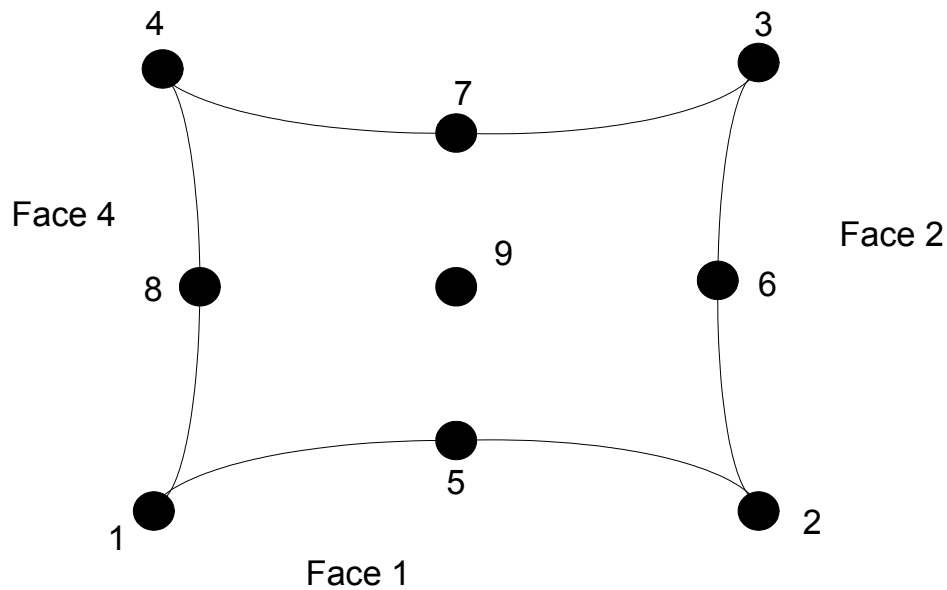


Figure 15 Order of Numbering for a 9-Node Element

4.0 FINITE ELEMENT MODEL

The current study involves a numerical investigation of the effects of systematically altering the dimensions of a WTEE beam. The WTEE beams are analyzed through the use of the commercial finite element software package ABAQUS. This section will describe in detail the ABAQUS models used for the parametric studies outlined in Section 5.0.

In the current research, WTEE beams are discretized into a number of nodes and shell elements. The actual number of elements and nodes is dependent upon the beam dimensions. A constant moment loading is achieved by applying a line load across the flange width at location corresponding to the mid-span of the simply supported beam configuration.

4.1 Geometry of Finite Element Model

The numerical model used is a representation of a simply supported singly symmetric prismatic WTEE beam subjected to a load at the midspan, which produces a constant moment. The only dimension that remains constant throughout the entirety the present parametric study is the depth of the beam. Other dimensions of the beam are varied in order to view the effects of the slenderness ratios, height of the beam to the thickness of the web, h/t_w , and the width of the flange to depth of beam, b/d on the structural ductility of fully braced WTEE beams subjected to constant moment loading. The actual dimensions will be discussed in detail in Section 5.

The distance between the two supports, L , is split into three equal lengths. The two end lengths are assigned properties that essentially make them behave as rigid ends. In order to accomplish this, the modulus of elasticity, E , in the shell elements comprising the ends is

increased to be 10 times that of ordinary steel. It is at the junctions of the rigid steel and the flexible middle section that the concentrated loads are applied though the imposition of a series of nodal loads applied along a line parallel to the flange width. In addition, at this location, the stem is constrained against out of plane translation. The AISC LRFD Equation F1-17 is employed to determine the length of each section in order that the effects of lateral-torsional buckling may be avoided within the context of the present study. For singly symmetric I-shaped members with the compression flange equal to or larger than the tension flange loaded in the plane of the web, the unbraced length for design by plastic analysis is defined as (AISC LRFD Third Edition):

$$L_{pd} = 0.12 + 0.076 \left(\frac{M_1}{M_2} \right) \left(\frac{E}{F_y} \right) r_y \quad (4-1)$$

Where:

$$\frac{M_1}{M_2} = -1.0 \quad (4-2)$$

The boundary conditions defined for the WTEE beams considered are representative of a simply supported beam. The flange is pinned along the top flange on one side, meaning that movement is restricted along the x, y, and z-axes. The other end of the flange has boundary conditions simulating a roller, i.e. movement along the y and z-axes is restricted. The only additional boundary conditions imposed upon the WTEE beam are those restricting out of plane bending at the junction of the rigid ends and the elastic portion of the beam.

The loads imposed on the beam are applied in the positive z direction. A unit load is applied to each node along the flange width at the rigid-elastic junction of the beam. The reader is referred to Figure 16 for clarification of the boundary conditions, applied loading and definition of the axes.

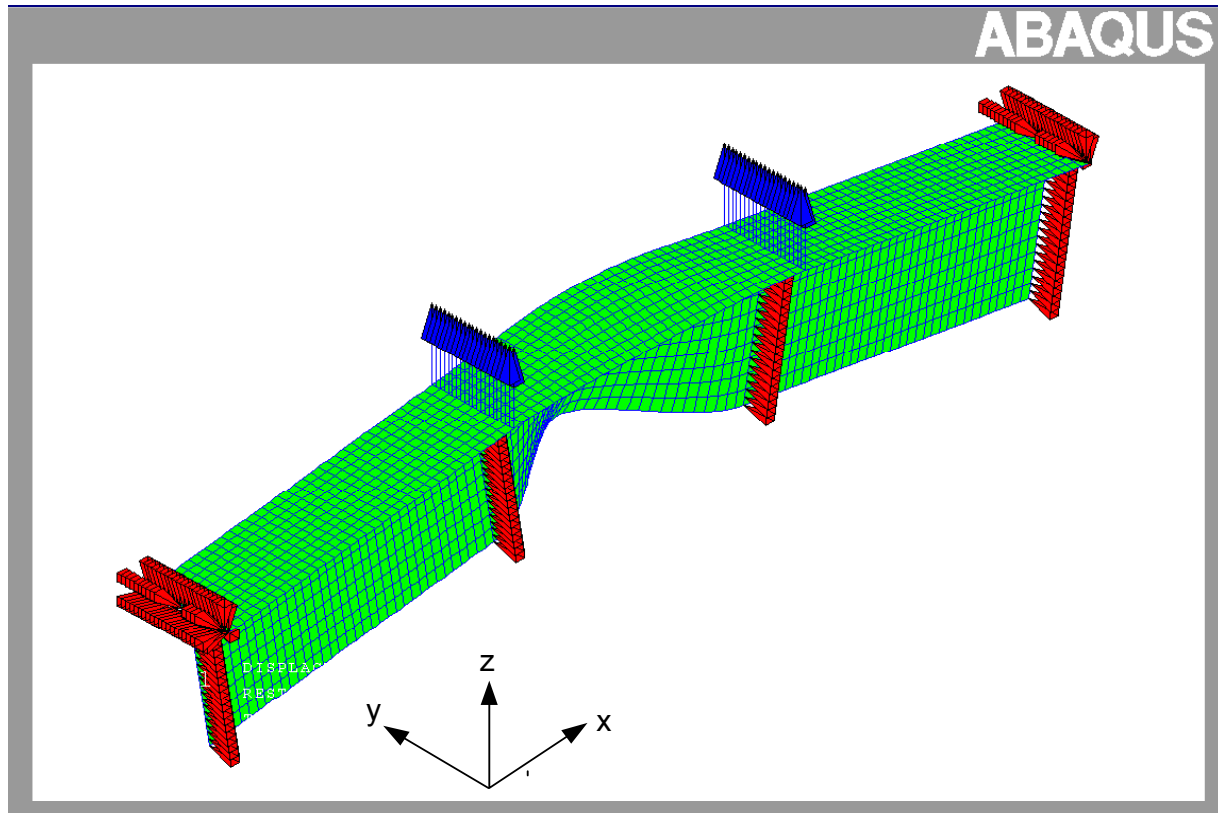


Figure 16 WTEE Beam Model

4.2 Finite Element Mesh

The WTEE beam numerical model constructed for this study is built from a dense finite element mesh of the ABAQUS S9R5 shell element described in Section 3.2. The mesh size is selected in order to maximize accuracy, while maintaining a reasonable computation time. A relationship exists between these two parameters; as accuracy of the model is increased through the introduction of more elements, the time required for ABAQUS to analyze the model is also increased. Figure 17 is an illustration of the mesh surface planes.

An aspect ratio of 1 was maintained for each of the elements of the WTEE beam model. That is, the sides of each of the elements are the same length. The planes of the mesh surfaces correspond with the middle surfaces of the constituent cross-sectional plate components of the beam as shown in Figure 17.

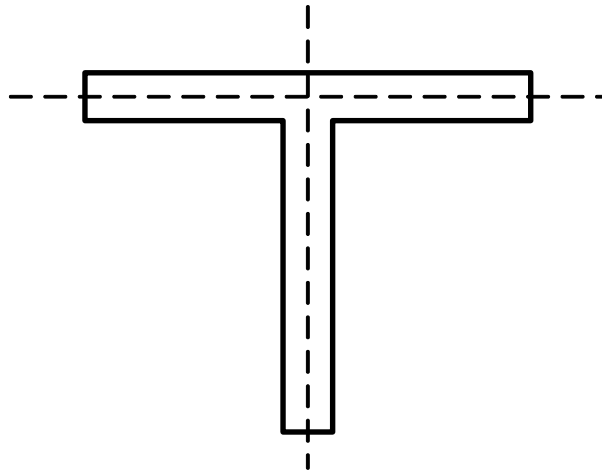


Figure 17 Mesh Surface Planes

Each of the cross-sectional plates is comprised of a node set. These node sets are then used to form an element set. Each element set may have its own properties. This allows for each of the plates to have a different thickness, which is imperative in the parametric studies considered herein. When the thickness of the flange, t_f , is altered, the height, h , of the WTEE is modified accordingly. The overall depth of the WTEE remains constant, however the height of the stem will vary due to the thickness of the flange.

The fact that the elements of the flange and stem are the same allows the two plate components to be compatible. This means that the stem mesh can be integrated with flange mesh through the use of an element set along the longitudinal centerline of the flange.

4.3 Imperfection Seed

In modeling studies where inelastic buckling is investigated, it is important that the evolution of the modeling solution be carefully monitored so that any indication of bifurcation in the equilibrium path is carefully assessed in order to try and ensure that the equilibrium branch being followed corresponds to the lowest energy state of the system (Earls and Shaw, 2001). Seeding the finite element mesh with an initial displacement field helps to ensure that the lowest energy path will be taken. The initial displacement is determined by using a linearized eigenvalue buckling analysis. From this analysis, an approximation to the critical buckling mode of the beam is obtained. In the current analysis, it was necessary to view each of the initial buckling modes determined by ABAQUS. Multiple buckling modes were produced for each imperfection study. However, the first buckling mode was not always the correct mode to use. It was necessary to look at each of the files and select an initial buckling mode that caused the stem

to have a slight buckle produced. It was common for the shorter length beams to have a first buckling mode in which the flange buckled first. The desired displacement field obtained from the linearized eigenvalue buckling analysis is then superimposed on the finite element model as a seed imperfection. This imperfection is scaled so that the maximum initial displacement anywhere in the mesh is equal to one-one-thousandth of the unbraced length determined from the AISC LRFD Equation F1-17, ($L_b/1000$). Upon imposition of the initial displacement field, a full, nonlinear finite element analysis can be carried out in order to observe subsequent WTEE flexural ductility.

4.4 Material Property Definitions

A homogeneous WTEE beam is used for this study, meaning that the same steel properties are used for each of the plates making up the WTEE section. The section is made up of 50 ksi steel. ABAQUS requires that material properties for finite-strain calculations be given in terms of true stress (force per current area) and logarithmic strain. The true stress (σ_{true}) and logarithmic plastic strain (ϵ_{ln}^{pl}) are expressed in terms of engineering stress and strain, respectfully as:

$$\sigma_{true} = \sigma_{eng} (1 + \epsilon_{eng}) \quad (4-3)$$

$$\epsilon_{ln}^{pl} = \ln(1 + \epsilon_{eng}) - \frac{\sigma_{true}}{E} \quad (4-4)$$

The input file for the study contains both elastic and plastic section properties for the beam. The elastic properties consist of the modulus of elasticity, E , and poisson's ratio, ν . For the current study, the modulus of elasticity is equal to 29000 ksi for the center length of the beam and 290000 ksi for the rigid ends, and poisson's ratio is kept constant at 0.3 for the entire length of the specimen. The plastic properties are given as points along the true stress versus true strain (logarithmic strain) curve as shown in Figure 18. ABAQUS uses these uniaxial material properties to extrapolate a yield surface in three-dimensional principal stress space using the von Mises yield criterion.

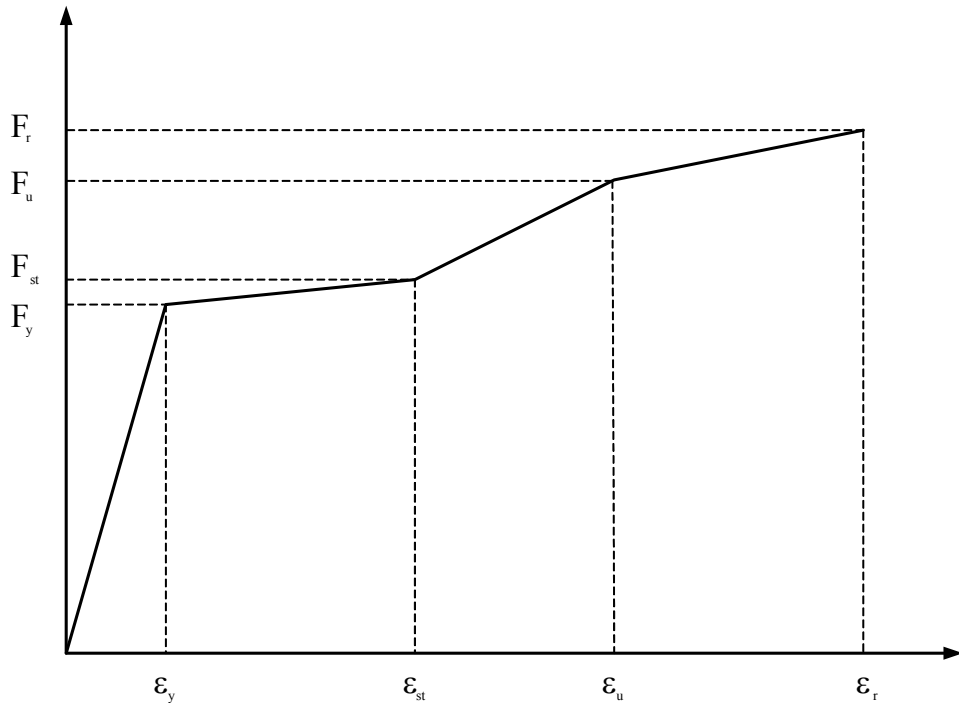


Figure 18 True Stress versus True Strain (Logarithmic Strain)

5.0 PARAMETRIC STUDIES AND RESULTS

The numerical model of Section 4 is used throughout the current parametric studies. The goal of this parametric study is to determine any correlations between the cross-sectional dimensions of the WTEE beam and its subsequent flexural ductility under the action of a constant moment loading.

The WTEE beam is put into negative flexure (stem in compression) by subjecting the beam to a constant moment through the application of a line load across the width of the flange at the one-third points of the beam, as illustrated in Figure 16.

The cross-sectional parameters varied in the present investigation are: the ratio b_f / d ; the flange slenderness ratio, $b_f / 2t_f$; and the web slenderness ratio, h/t_w . The results from the study of various parametric combinations involving the foregoing are studied in order that conclusions with regards to the relationships between the cross-sectional dimensions of the beams and the flexural ductility of the beam may be identified.

5.1 Parametric Studies

Eight different b_f / d ratios are explored, (1) 2.0, (2) 1.8, (3) 1.6, (4) 1.4, (5) 1.2, (6) 1.0, (7) 0.8, and (8) 0.6. To begin, the thickness of the flange is defined using the AISC LRFD provisions of Table B5.1, “Limiting Width-Thickness Ratios for Compression Elements”. This was done in order to define the thickness of the flange required to produce a compact section (as if the flange of the WTEE were in flexural compression). The value of λ_p , the requirement for a compact width-thickness ratio, is taken to be the case of I-shaped rolled beams and channels in flexure. This can be expressed as:

$$\lambda_p = \frac{65}{\sqrt{F_y}} \quad (5-1)$$

To fulfill the slenderness equation,

$$\frac{b_f}{2t_f} = \lambda_p \quad (5-2)$$

With the depth, d , held constant at 14”, the width to depth ratio, b_f / d , was used to define the width of the flange, b_f . This dimension was then used in the width to thickness ratio of equation (5-2) to determine the thickness of the flange, t_f . As stated in Section 4, the length of the beam was made to be three times the unbraced length, L_{pd} , as defined in Chapter F of the AISC LRFD.

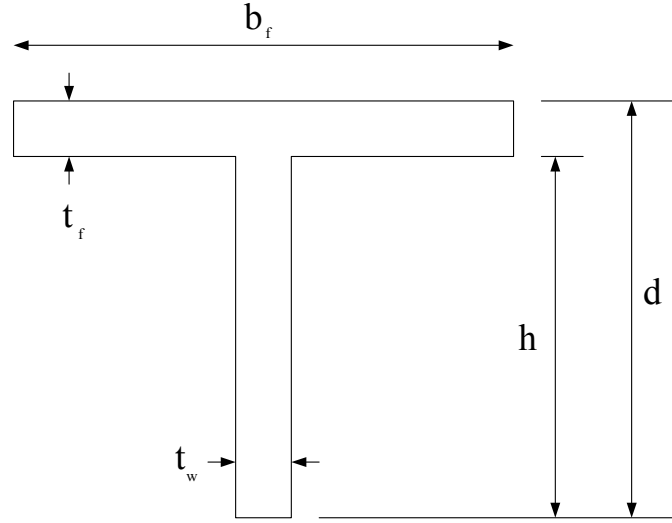


Figure 19 WTEE Beam Dimensions

In addition to the foregoing parametric variations, the thickness of the web, t_w , was also systematically varied throughout the study in order that the flexural ductility of the beam, observed via the rotation capacity, could be studied. With all of the basic beam cross-sectional dimensions defined as in Figure 19, the parametric studies are conducted through the use of ABAQUS.

An individual study was conducted for each of the eight b_f/d ratios identified. Taking, as an example, the trial where $b_f/d = 1.8$, the width of the flange is defined as $b_f = 1.8(d)$ where $d = 14"$. The thickness of the flange is determined based upon the ratio from equation (5-2). From the value of b_f as determined earlier, this ratio is subsequently used to calculate the thickness of the flange:

$$t_f = \frac{b_f}{65(2)} \quad (5-3)$$

Initially, the thickness of the web is given an assumed value. Using this value, the beam is analyzed and the normalized moment-rotation response is plotted in order to obtain the beam rotation capacity. Based upon the results of this analysis, the thickness of the web is either increased or decreased. This process is repeated until a thickness of the web is determined that will yield a moment rotation of approximately 3.0.

In order to draw conclusions in regards to the behavior of the beam when the thickness of the web approaches that necessary to produce an R-value of approximately 3.0, various parameters were recorded and tabulated. These results are shown in Section 5.2.

5.2 Presentation of Results

The results of the first three trials of the parametric studies are presented in Table 1. That is, the thickness of the web is varied while maintaining flange compactness, as per equation (5-1), for a given b/d ratio. Column 1 is an indication as to which trial is being analyzed. The span to depth ratio, L_{pd}/d , is also tabulated for later observations and conclusions. The ratio, $b_f/2t_f$, is constant because, for these cases, equation (5-2) is satisfied. The values tabulated in the fourth column, h/t_w , the ratio of the height of the WTEE beam to the thickness of the web, is also tabulated for later observations and conclusions. M_u/M_p is the ratio of the maximum moment that the beam reached as compared with the calculated plastic moment of the beam. The last column is a tabulation of R , the rotation capacity, for each of the considered parametric combinations on the given row.

Table 1 Results of the First Three Trials

$\frac{b_f}{d}$	$\frac{L_{pd}}{d}$	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	$\frac{M_u}{M_p}$	R
0.6	2.128	9.190	13.543	0.749	-
0.6	2.014	9.190	11.286	0.782	-
0.6	1.934	9.190	9.674	0.851	-
0.6	1.882	9.190	8.464	0.946	-
0.6	1.853	9.190	7.524	1.045	11.635
0.6	1.843	9.190	6.772	1.119	17.115
0.8	3.451	9.195	13.391	0.784	-
0.8	3.261	9.195	11.159	0.792	-
0.8	3.109	9.195	9.565	0.801	-
0.8	2.988	9.195	8.369	0.841	-
0.8	2.892	9.195	7.439	0.900	-
0.8	2.817	9.195	6.696	0.996	-
1.0	5.226	9.198	16.549	0.895	-
1.0	4.936	9.198	13.239	0.903	-
1.0	4.810	9.198	12.035	0.901	-
1.0	4.694	9.198	11.033	0.889	-
1.0	4.588	9.198	10.184	0.875	-

Table 2 summarizes the results from the remaining trials. The same ratios are tabulated for these cases.

Table 2 Parametric Studies with Acceptable Span to Depth Ratio

$\frac{b_f}{d}$	$\frac{L_{pd}}{d}$	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	$\frac{M_u}{M_p}$	R
1.2	6.657	9.190	14.540	0.978	-
1.2	6.580	9.190	13.775	1.010	1.327
1.2	6.535	9.190	13.353	1.026	1.857
1.2	6.505	9.190	13.086	1.031	3.504
1.2	6.232	9.190	10.905	1.161	5.261
1.4	8.112	9.193	12.934	1.012	1.644
1.4	7.964	9.193	11.758	1.060	3.072
1.4	7.893	9.193	11.247	1.118	4.553
1.4	7.691	9.193	9.949	1.173	8.017
1.6	9.481	9.195	10.925	1.047	3.152
1.6	8.796	9.195	10.652	1.061	3.628
1.6	9.303	9.195	9.832	1.113	5.693
1.6	9.048	9.195	8.521	1.254	11.232
1.8	11.061	9.190	10.524	1.025	2.885
1.8	11.033	9.190	10.352	1.034	3.190
1.8	10.424	9.190	9.715	1.070	4.344
1.8	10.793	9.190	9.021	1.132	6.577
1.8	10.667	9.190	8.419	1.197	9.259
1.8	10.545	9.190	7.893	1.242	12.093
2.0	12.612	9.192	9.982	1.008	3.119
2.0	12.546	9.192	9.598	1.032	3.893
2.0	12.416	9.192	8.912	1.089	5.766
2.0	12.291	9.192	8.318	1.082	6.299

5.3 Increasing the Thickness of the Flange

The parametric studies were extended to include a study to explore the behavior of the same WTEE beam when the thickness of the flange was increased. The influence of the flange thickness was thought to be an important parameter for consideration since the flange-web junction of the WTEE may be thought of as being restrained by a linear spring whose stiffness is proportional to flange thickness. As a result, variation of the flange thickness ought to be considered in the study of cross-sectional parameters influencing WTEE flexural ductility.

For each of the eight variations of the ratio b_f / d considered, a final beam size was found that yielded a rotation capacity of approximately 3.0. It was these beams that were explored further within the context of this extension of the parametric study. This increase in the thickness of the flange was accomplished by altering the original slenderness ratio:

$$\frac{b_f}{2t_f} = \frac{65}{\sqrt{F_y}} \quad (5-4)$$

It is noted that the flange compactness given in equation (5-4) is for a flange in uniform compression; a condition that our WTEE flanges does not experience. The flange compactness criteria was used simply as a reasonable starting point from which to explore more slender tension flange alternatives. The first variation considered defined the flange slenderness ratio to be 20% greater than that necessary to produce a compact beam.

$$\frac{b_f}{2t_f} = 1.2 \frac{65}{\sqrt{F_y}} \quad (5-5)$$

The thickness of the flange was varied for this analysis while all other dimensions were held constant. The purpose of this alteration was to observe the resulting behavior, and determine if increasing the thickness of the flange by 20% would affect the flexural ductility of the beam.

For each of the trials as described in Table 2, in which a rotation capacity of approximately 3.0 was obtained, the model was altered as described above. Because none of the models of Table 1 had a rotation capacity of approximately 3.0, none of these cases were further explored. An explanation of these cases is presented in Section 5.4. The results for the cases from Table 2 are presented below in Table 3.

Table 3 Increase of t_f by 20%

$\frac{b_f}{d}$	$\frac{L_{pd}}{d}$	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	$\frac{M_u}{M_p}$	R
1.2	6.209	11.031	13.239	1.045	1.814
1.4	7.638	11.031	11.920	1.066	5.620
1.6	9.135	11.031	11.098	1.069	3.695
1.8	10.675	11.031	10.539	1.046	3.176
2.0	12.251	11.031	10.185	1.027	3.071

It is clear that for the cases where $b_f/d=1.4$ through 2.0, little change in the rotation capacity is observed. Noting that the value of the rotation capacity decreased for the case of $b_f/d=1.2$, further observations will be made to address this behavior in Section 5.4. More

importantly, the flexural ductility of the beam is not decreased. Therefore, from these results, it can be observed that it is only necessary to fulfill the original slenderness ratio of equation (5-4).

A further increase in the thickness of the flange is explored to validate these observations. The ratio was then changed in order to increase equation (5-4) by 40%:

$$\frac{b_f}{2t} = 1.4 \frac{65}{\sqrt{F_y}} \quad (5-6)$$

Again, the flexural ductility of the beam was observed through the calculation of the rotation capacity to determine if an increase in the thickness of the flange by 40% of the original thickness would affect the flexural ductility of the beam. The cases from Table 2 are presented below in Table 4.

Table 4 Increase of t_f by 40%

$\frac{b_f}{d}$	$\frac{L_{pd}}{d}$	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	$\frac{M_u}{M_p}$	R
1.2	5.950	12.869	13.347	0.913	-
1.4	9.349	12.869	12.035	1.047	2.916
1.6	8.825	12.869	11.222	1.076	3.867
1.8	10.350	12.869	10.673	1.052	3.394
2.0	11.918	12.869	10.330	1.042	3.259

It is clear that for the cases of b_f/d equal to 1.4 through 2.0, this change has little effect on the rotation capacity. It is only necessary to note that there is no decrease in rotation capacity in these cases. However, for the instance of b_f/d equal to 1.2, a decrease in the rotation capacity is observed.

These results support the theory that the flange thickness need only satisfy equation (5-4). This, along with a sufficient web thickness will produce a compact beam.

5.4 Span to Depth Ratio

It was noted that the variations presented in Table 1 were not able to achieve a rotation capacity of approximately 3.0. In addition, when the thickness of the flange was increased, the case of $b_f/d=1.2$, showed a decrease in the rotation capacity of the beam.

It was then observed from the ratios tabulated in these corresponding tables that the span to depth ratio was quite small; below 6.5. In these particular cases, the stress in the beam reached and exceeded the yield stress of the material as evidenced by viewing the von-Mises stresses of the beam. Figure 19 illustrates the von Mises stress distribution in a representative WTEE beam with low span-to-depth ratio. In these cases, the member is no longer conforming to Bernoulli-Euler beam behavior; the span to depth ratio is too small. As a result, the cross sectional material reaches yield prior to the attainment of sufficient longitudinal stress intensities, needed for attainment of M_p , as a result of the presence of large shear forces. From the parametric studies conducted throughout this section, it appears that a practical span-to-depth ratio needed for the attainment of M_p might be 6.5.

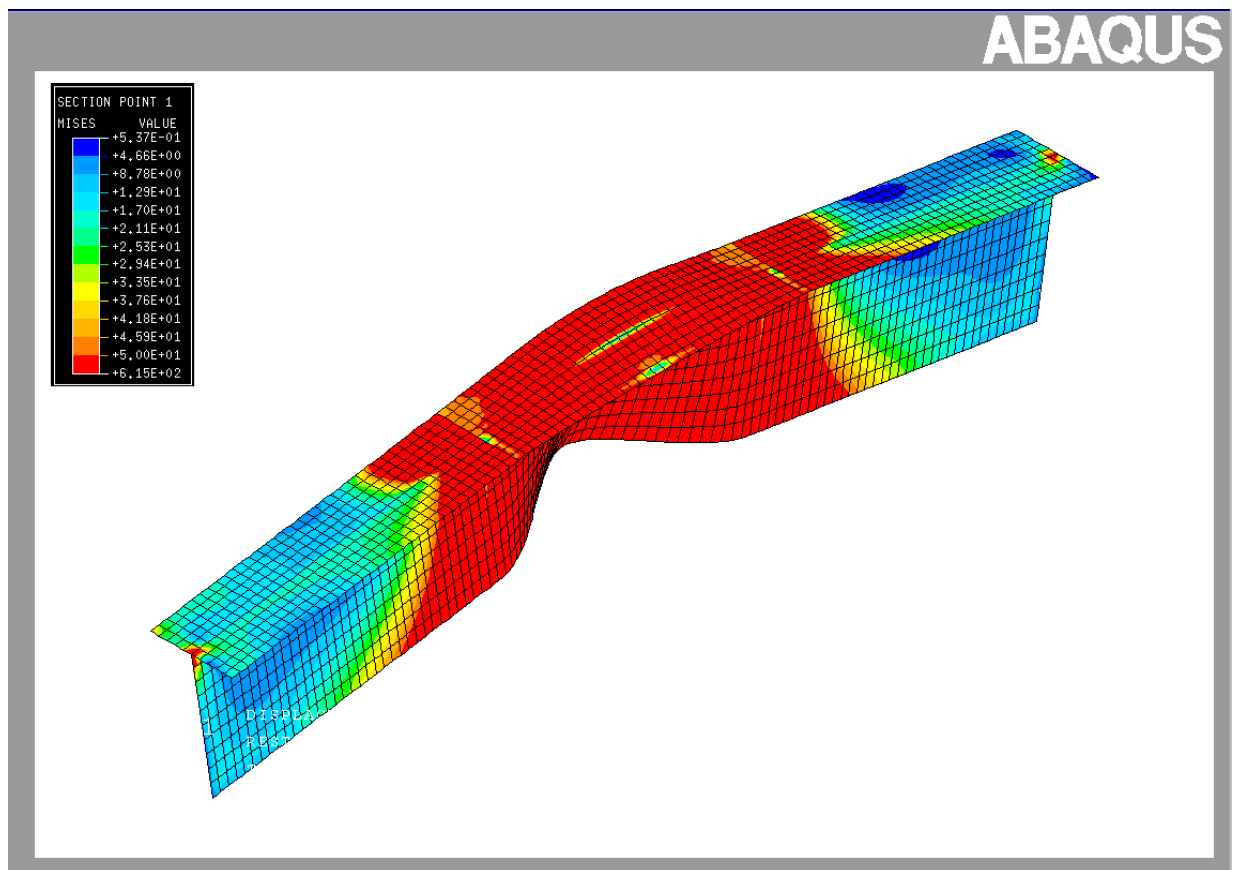


Figure 20 Failure Mode of WTEE Beam with a Low Value of L_{pd} / d

6.0 CONCLUSIONS

From the current research, it has been determined that correlations exist between the WTEE beam dimensions and the flexural ductility of the beam. This section will provide a summary of the requirements that must be satisfied in order to obtain a compact beam. These requirements are presented in the form of plate slenderness ratios.

The ratio of the height to the thickness of the web, h/t_w , followed a pattern throughout the current research. From the tables presented in Section 5, it is apparent that this ratio must be approximately equal to 10. $\frac{h}{t_w} \cong 10$

Additionally, for a WTEE beam subjected to negative flexure (stem in compression), the rotation capacity does not appear to be sensitive to the flange slenderness ratio, $b_f / 2t_f$, within a range that is consistent with hot-rolled steel member proportions.

Before the two previously enumerated plate slenderness requirements may be implemented, it is required that the beam satisfy a limitation on the span to depth ratio, L_{pd} / d . It has been determined through the parametric studies described in Section 5, and the explanation of Section 5.4, that this ratio must be greater than 6.5. If this requirement is not satisfied, high transverse shear forces may interfere with development of the full plastic capacity of the cross-section. $\frac{L_{pd}}{d} > 6.5$

Further research in this area would be beneficial in the development of additional applicable equations and limiting ratios.

APPENDIX A

Input for ABAQUS

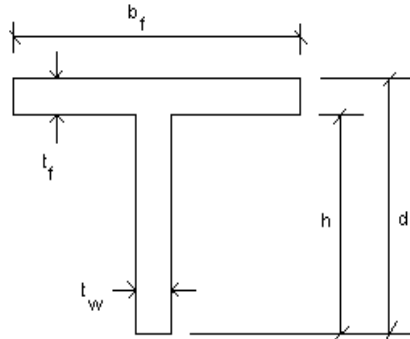
Prior to the WTEE being modeled in ABAQUS, certain properties and dimensions must be determined in order to create an accurate model. The first section of this appendix, A1, is a worksheet used to calculate the section properties of any particular WTEE shape. The original format for this file was MathCAD. The properties and dimensions calculated in this worksheet were inputted into an excel spreadsheet. This spreadsheet, presented in section A2 of this appendix, was designed to create an input file for ABAQUS. The spreadsheet was written in such a way that minimal alterations were necessary for each change of beam dimensions. The formulas involved in this spreadsheet are presented first, followed by an actual spreadsheet. Upon the completion of this step, an input file is easily created. A typical seed imperfection file and a material property file for one beam are presented to illustrate how each of the beams were inputted into ABAQUS. For Appendix A, the WTEE beam used as an illustrative example was that of $b_f / d = 2.0$ and $t_w = 1.25$, using the ratio, $b_f / 2t_f = 65 / \sqrt{F_y}$.

Appendix A1: Section Properties

Cross sectional properties for WTEE:

$$b_f/d=2$$

$$\text{var2_}1.25$$



Assigned Properties:

$$d := 14$$

$$E := 29000$$

$$G := 11200$$

$$F_y := 50$$

$$b_f := 2 \cdot d \quad b_f = 28$$

$$t_f := \frac{b_f \cdot \sqrt{F_y}}{65.2}$$

$$t_w := 1.25$$

$$h := d - t_f$$

$$h = 12.477$$

$$t_f = 1.523$$

$$\frac{h}{t_w} = 9.982$$

Calculations of Properties:

$$A1 := b_f \cdot t_f$$

$$A2 := h \cdot t_w$$

$$A1 = 42.644$$

$$A2 = 15.596$$

Centroids measured from the bottom of the stem:

$$y1 := \left(d - \frac{t_f}{2} \right)$$

$$y2 := \frac{h}{2}$$

$$y1 = 13.239$$

$$y2 = 6.239$$

Location of the Neutral Axis:

$$\bar{y} := \frac{A1 \cdot y1 + A2 \cdot y2}{A1 + A2}$$

$$\bar{y} = 11.364$$

Moment of Inertia about the X axis:

$$I_x := \frac{tw \cdot (h)^3}{12} + [A2 \cdot (ybar - y2)^2] + \frac{bf \cdot tf^3}{12} + A1 \cdot (y1 - ybar)^2$$

$$I_x = 770.138$$

$$I_y := \frac{(d - tf) \cdot tw^3}{12} + \frac{tf \cdot (bf)^3}{12}$$

$$I_y = 2.788 \times 10^3$$

Radius of Gyration:

$$r_x := \sqrt{\frac{I_x}{A1 + A2}}$$

$$r_y := \sqrt{\frac{I_y}{A1 + A2}}$$

$$r_x = 3.636$$

$$r_y = 6.919$$

Section Modulus:

$$Cfl := d - ybar$$

$$Cst := ybar$$

$$Cw := 0$$

$$Sfl := \frac{I_x}{Cfl}$$

$$Sst := \frac{I_x}{Cst}$$

Polar Moment of Inertia:

$$J := \frac{bf \cdot (tf)^3}{3} + \frac{(d - tf) \cdot (tw)^3}{3}$$

First Calculation of Plastic Section Modulus

Location of the Plastic Neutral Axis

$$Y1_a := \frac{bf \cdot tf - h \cdot tw}{bf} \quad Y2_a := \frac{h \cdot tw - bf \cdot tf}{tw}$$

Plastic Section Modulus

$$Z_a := \frac{(tf - Y1_a)^2}{2} \cdot bf + \left(\frac{Y1_a^2}{2} \cdot bf \right) + \frac{h^2}{2} \cdot tw$$

$$Z_b := bf \cdot tf \cdot \left(\frac{tf}{2} + Y2_a \right) + \left(\frac{tw^2}{2} \cdot Y2_a \right) + \frac{(h - tw)^2}{2} \cdot tw$$

$$Z1 := \text{if}(A2 < A1, Z_a, Z_b)$$

$$Z1 = 114.705$$

Second Calculation of Plastic Section Modulus

Location of the Plastic Neutral Axis:

$$Y1 := \frac{bf \cdot tf + tf \cdot tw - d \cdot tw}{2 \cdot bf} \quad Y2 := \frac{d \cdot tw - tf \cdot tw - bf \cdot tf}{2 \cdot tw}$$

$$Y1 = 0.483$$

$$Y2 = -10.819$$

Plastic Section Modulus:

$$Z1 := \left[\left(\frac{tf - Y1}{2} + Y1 \right) \cdot (tf - Y1) \cdot bf \right] + \left[\frac{Y1^2}{2} \cdot bf + \left[\left(\frac{d - tf}{2} \right) + Y1 \right] \cdot (d - tf) \cdot tw \right]$$

$$Z2 := bf \cdot tf \cdot \left(\frac{tf}{2} + Y2 \right) + \left[\frac{(d - tf - Y2)^2}{2} \cdot tw + \frac{Y2^2 \cdot tw}{2} \right]$$

$$Z2 := \text{if}(A2 < A1, Z1, Z2)$$

$$Z2 = 137.304$$

Final Plastic Section Modulus

$$Z := \frac{Z1 + Z2}{2}$$

$$Z = 126.004$$

Plastic Moment:

Calculated

$$Mp := Z \cdot Fy$$

$$Mp = 6.3 \times 10^3$$

Summary :

Areas:

$$A1 = 42.644 \quad A2 = 15.596 \quad A1 + A2 = 58.24$$

Slenderness Ratios:

$$\frac{bf}{2 \cdot tf} = 9.192 \quad \frac{h}{tw} = 9.982 \quad \frac{tw}{2} = 0.625$$

Moments of Inertia and Radius of Gyration:

$$I_x = 770.138 \quad r_x = 3.636 \quad I_y = 2.788 \times 10^3 \quad r_y = 6.919$$

Elastic Section Moduli:

$$S_{st} = 67.77 \quad S_{fl} = 292.157 \quad J = 41.094$$

Plastic Section Modulus:

$$Z = 126.004$$

Unbraced Length:

$$L_b := [0.12 + 0.076 \cdot (-1)] \cdot \left(\frac{E}{F_y} \right) \cdot r_y \quad L_b = 176.573$$

Plastic Moment:

$$M_p := Z \cdot F_y \quad M_p = 6.3 \times 10^3$$

Yield Moment:

$$M_y := F_y \cdot S_{st} \quad M_y = 3.389 \times 10^3$$

Maximum Deflection:

$$\Delta_{\max} := \frac{M_p}{24 \cdot E \cdot I_x} \cdot [3 \cdot (3 \cdot L_b)^2 - 4 \cdot (L_b)^2] \quad \Delta_{\max} = 8.429$$

Rotation:

$$\theta_p := \frac{M_p \cdot L_b}{2 \cdot E \cdot I_x}$$

$$\theta_p = 0.0249$$

Span to Depth Ratio:

$$\frac{L_b}{d} = 12.612$$

Appendix A2: Development Formulas

Filename		var2_1.25	
*Heading			
alter tw while bf/d=2.0			
RIGID ENDS			
SRRS ELEMENTS			
APPROXIMATELY 1" ELEMENTS			
ACTUAL WT EE DIMENSIONS			
**			
**INITIAL CONDITIONS			
**			
INPUT VARIABLES			
		INCHES	STEEL
		Lb= 176.573	KSI
		L= ~3"C19	E= 29000
		bf= 28	Fy= 50
		d= 14	v= 0.3
		tf= 1.523	RIGID
		tw= 1.25	E= 290000
NUMBER OF ELEMENTS IN FLANGE		10	THIS NUMBER CAN E
NUMBER OF ELEMENTS IN LENGTH		=C19/E29	
FLANGE ELEMENT SIZE		=C21/E26	midpoint
LENGTH ELEMENT SIZE		=C19/(ROUND(E27.0))	=((A49+A51)/2)+((E40+1)/2)
WEB ELEMENT SIZE		1.4125	
MESH SETUP			
		EL FL	=C21/E29
		EL Lb	=C19/E30
		EL WB	=C22/E31-1
		STITCH	=C22-(E31"E37)
		FLANGE LENGTH INC	=E35"2+1
		FLANGE INC	=E40-1
		WEB LENGTH INC	=E37"2+1
		WEB INC	=E42-1
INITIAL COORIDINATES			
*NODE			
1	0	0	0
=E35"2+1	0	=E29"E35	0
=2"E36"E40+A47	=E30"E36	0	0
=A49+E41	=E30"E36	=E29"E35	0
=2"E36"E40+A49	=2"E30"E36	0	0
=A51+E41	=2"E30"E36	=E29"E35	0
=2"E36"E40+A51	=3"E30"E36	0	0
=A53+E41	=3"E30"E36	=E29"E35	0
=A54+1	0	=(E29"E35)/2	=(E31"E37)+E38)
=A55+E43	0	=(E29"E35)/2	=E38
=2"E36"E42+A55	=E30"E36	=(E29"E35)/2	=(E31"E37)+E38)
=A57+E43	=E30"E36	=(E29"E35)/2	=E38
=2"E36"E42+A57	=2"(E30"E36)	=(E29"E35)/2	=(E31"E37)+E38)
=A59+E43	=2"(E30"E36)	=(E29"E35)/2	=E38
=2"E36"E42+A59	=3"(E30"E36)	=(E29"E35)/2	=(E31"E37)+E38)
=A61+E43	=3"(E30"E36)	=(E29"E35)/2	=E38
51000	=0	=(E29"E35)/2	=(E38/2)
=2"3"E36+A63)	=3"(E30"E36)	=(E29"E35)/2	=(E38/2)

First node of stitch, sta must be greater than la	
---	--

```

**
**NODE DEFINITION
**
*NGEN          NSET=BFLANGELT
=A47            =A49              =E40
*NGEN          NSET=TFLANGELT
=A48            =A50              =E40
*NGEN          NSET=BFLANGEMID
=A49            =A51              =E40
*NGEN          NSET=TFLANGEMID
=A50            =A52              =E40
*NGEN          NSET=BFLANGERT
=A51            =A53              =E40
*NGEN          NSET=TFLANGERT
=A52            =A54              =E40
*NGEN          NSET=BWEBLT
=A53            =A57              =E42
*NGEN          NSET=TWEBLT
=A56            =A58              =E42
*NGEN          NSET=BWEBMID
=A57            =A59              =E42
*NGEN          NSET=TWEBMID
=A58            =A60              =E42
*NGEN          NSET=BWEBRT
=A59            =A61              =E42
*NGEN          NSET=TWEBRT
=A60            =A62              =E42
*NGEN          NSET=STITCH
=A63            =A64              =1

**
**FILL NODES
**

*NFILL          NSET=FLANGELT
BFLANGELT      TFLANGELT          =E41
*NFILL          NSET=FLANGEMID
BFLANGEMID     TFLANGEMID         =E41
*NFILL          NSET=FLANGERT
BFLANGERT      TFLANGERT          =E41
*NFILL          NSET=WEBLT
BWEBLT         TWEBLT             =E43
*NFILL          NSET=WEBMID
BWEBMID        TWEBMID            =E43
*NFILL          NSET=WEBRT
BWEBRT         TWEBRT             =E43

**
**CONSTRAINT SETS
**

*NSET           NSET=FLPINLT      GENERATE
=A47            =A48              1
*NSET           NSET=FLPINRT      GENERATE
=A53            =A54              1
*NSET           NSET=WEBPINLT     GENERATE
=A55            =A56              1
*NSET           NSET=WEBPINRT     GENERATE
=A61            =A62              1
*NSET           NSET=MIDPINLT     GENERATE
=A57            =A58              1
*NSET           NSET=MIDPINRT     GENERATE
=A59            =A60              1
*NSET           NSET=LTLOAD       GENERATE
=A49            =A50              1
*NSET           NSET=RTLOAD       GENERATE
=A51            =A52              1

```

**									
**FLANGELT									
**									
*ELEMENT	MASTER ELEMENT								
1	TYPE=S9R5								
	=A47	=(A47+E40)+E40	=C139+2	=B139+2	=B139+E40	=C139+1	=F139+2	=B139+1	=F139+1
	GENERATE ELEMENT SET								
*ELGEN	ELSET=FLANGELT								
=A139	=E\$36	=A\$47+E\$40+E\$41		=E\$35	=2	=(3*E\$36)			
**									
**FLANGEMID									
**									
*ELEMENT	TYPE=S9R5								
=E36+1	=A49	=(A49+E40+E40)	=C147+2	=B147+2	=B147+E40	=C147+1	=E147+E40	=B147+1	=F147+1
*ELGEN	ELSET=FLANGEMID								
=A147	=E\$36	=A\$47+E\$40+E\$41		=E\$35	=2	=(3*E\$36)			
**									
**FLANGERT									
**									
*ELEMENT	TYPE=S9R5								
=2*E36+1	=A51	=A51+E40+E40	=C154+2	=B154+2	=B154+E40	=C154+1	=E154+E40	=B154+1	=F154+1
*ELGEN	ELSET=FLANGERT								
=A154	=E\$36	=A\$47+E\$40+E\$41		=E\$35	=2	=(3*E\$36)			
**									
**WEBLT									
**									
*ELEMENT	TYPE=S9R5								
=(3*E36)+E35+1	=A55	=A55+E42+E42	=C161+2	=B161+2	=B161+E42	=C161+1	=E161+E42	=B161+1	=F161+1
*ELGEN	ELSET=WEBLT								
=A161	=E\$36	=E\$42+E\$42	1	=E\$37	=2	=(3*E\$36)			
**									
**WEBMID									
**									
*ELEMENT	TYPE=S9R5								
=E36+A163	=A57	=A57+E42+E42	=C168+2	=B168+2	=A57+E42	=C168+1	=E168+E42	=B168+1	=F168+1
*ELGEN	ELSET=WEBMID								
=A168	=E\$36	=C163	1	=E\$37	=2	=(3*E\$36)			
**									
**WEBRT									
**									
*ELEMENT	TYPE=S9R5								
=A170+E36	=A59	=A59+E42+E42	=C175+2	=B175+2	=B175+E42	=C175+1	=E175+E42	=B175+1	=F175+1
*ELGEN	ELSET=WEBRT								
=A175	=E\$36	=C163	1	=E\$37	=2	=(3*E\$36)			

*ELEMENT
=3*E36*E37+A161

*ELSET	ELSET=STITCHLT
=A182	=A244

```

**
**STITCHMID
**
*ELEMENT                                     TYPE=S9R5
=A182+E36                                  =A58
=A251+1                                    =C251
=A252+1                                    =C252
=A253+1                                    =C253
=A254+1                                    =C254
=A255+1                                    =C255
=A256+1                                    =C256
=A257+1                                    =C257
=A258+1                                    =C258
=A259+1                                    =C259
=A260+1                                    =C260
=A261+1                                    =C261
=A262+1                                    =C262
=A263+1                                    =C263
=A264+1                                    =C264
=A265+1                                    =C265
=A266+1                                    =C266
=A267+1                                    =C267
=A268+1                                    =C268
=A269+1                                    =C269
=A270+1                                    =C270
=A271+1                                    =C271
=A272+1                                    =C272
=A273+1                                    =C273
=A274+1                                    =C274
=A275+1                                    =C275
=A276+1                                    =C276
=A277+1                                    =C277
=A278+1                                    =C278
=A279+1                                    =C279
=A280+1                                    =C280
=A281+1                                    =C281
=A282+1                                    =C282
=A283+1                                    =C283
=A284+1                                    =C284
=A285+1                                    =C285
=A286+1                                    =C286
=A287+1                                    =C287
=A288+1                                    =C288
=A289+1                                    =C289
=A290+1                                    =C290
=A291+1                                    =C291
=A292+1                                    =C292
=A293+1                                    =C293
=A294+1                                    =C294
=A295+1                                    =C295
=A296+1                                    =C296
=A297+1                                    =C297
=A298+1                                    =C298
=A299+1                                    =C299
=A300+1                                    =C300
=A301+1                                    =C301
=A302+1                                    =C302
=A303+1                                    =C303
=A304+1                                    =C304
=A305+1                                    =C305
=A306+1                                    =C306
=A307+1                                    =C307
=A308+1                                    =C308
=A309+1                                    =C309
=A310+1                                    =C310
=A311+1                                    =C311
=A312+1                                    =C312
*ELSET                                     ELSET=STITCHMID
=A251                                     =A313
                                     1

=B251+E42+E42      =E251+E40+E40      =E182+2"E36"E40      =B251+E42      =J244+3      =E251+E40      =G251-2      =I251+1      =IF(C251=$A$60,"STOP","")
=B252+E$42+E$42    =D252+E$40+E$40    =D252      =B252+E$42    =G252+2      =E253+E$40    =G252      =I253+1      =IF(C252=$A$60,"STOP","")
=B253+E$42+E$42    =D253+E$40+E$40    =D253      =B253+E$42    =G253+2      =E254+E$40    =G253      =I254+1      =IF(C253=$A$60,"STOP","")
=B254+E$42+E$42    =D254+E$40+E$40    =D254      =B254+E$42    =G254+2      =E255+E$40    =G254      =I255+1      =IF(C254=$A$60,"STOP","")
=B255+E$42+E$42    =D255+E$40+E$40    =D255      =B255+E$42    =G255+2      =E256+E$40    =G255      =I256+1      =IF(C255=$A$60,"STOP","")
=B256+E$42+E$42    =D256+E$40+E$40    =D256      =B256+E$42    =G256+2      =E257+E$40    =G256      =I257+1      =IF(C256=$A$60,"STOP","")
=B257+E$42+E$42    =D257+E$40+E$40    =D257      =B257+E$42    =G257+2      =E258+E$40    =G257      =I258+1      =IF(C257=$A$60,"STOP","")
=B258+E$42+E$42    =D258+E$40+E$40    =D258      =B258+E$42    =G258+2      =E259+E$40    =G258      =I259+1      =IF(C258=$A$60,"STOP","")
=B259+E$42+E$42    =D259+E$40+E$40    =D259      =B259+E$42    =G259+2      =E260+E$40    =G259      =I260+1      =IF(C259=$A$60,"STOP","")
=B260+E$42+E$42    =D260+E$40+E$40    =D260      =B260+E$42    =G260+2      =E261+E$40    =G260      =I261+1      =IF(C260=$A$60,"STOP","")
=B261+E$42+E$42    =D261+E$40+E$40    =D261      =B261+E$42    =G261+2      =E262+E$40    =G261      =I262+1      =IF(C261=$A$60,"STOP","")
=B262+E$42+E$42    =D262+E$40+E$40    =D262      =B262+E$42    =G262+2      =E263+E$40    =G262      =I263+1      =IF(C262=$A$60,"STOP","")
=B263+E$42+E$42    =D263+E$40+E$40    =D263      =B263+E$42    =G263+2      =E264+E$40    =G263      =I264+1      =IF(C263=$A$60,"STOP","")
=B264+E$42+E$42    =D264+E$40+E$40    =D264      =B264+E$42    =G264+2      =E265+E$40    =G264      =I265+1      =IF(C264=$A$60,"STOP","")
=B265+E$42+E$42    =D265+E$40+E$40    =D265      =B265+E$42    =G265+2      =E266+E$40    =G265      =I266+1      =IF(C265=$A$60,"STOP","")
=B266+E$42+E$42    =D266+E$40+E$40    =D266      =B266+E$42    =G266+2      =E267+E$40    =G266      =I267+1      =IF(C266=$A$60,"STOP","")
=B267+E$42+E$42    =D267+E$40+E$40    =D267      =B267+E$42    =G267+2      =E268+E$40    =G267      =I268+1      =IF(C267=$A$60,"STOP","")
=B268+E$42+E$42    =D268+E$40+E$40    =D268      =B268+E$42    =G268+2      =E269+E$40    =G268      =I269+1      =IF(C268=$A$60,"STOP","")
=B269+E$42+E$42    =D269+E$40+E$40    =D269      =B269+E$42    =G269+2      =E270+E$40    =G269      =I270+1      =IF(C269=$A$60,"STOP","")
=B270+E$42+E$42    =D270+E$40+E$40    =D270      =B270+E$42    =G270+2      =E271+E$40    =G270      =I271+1      =IF(C270=$A$60,"STOP","")
=B271+E$42+E$42    =D271+E$40+E$40    =D271      =B271+E$42    =G271+2      =E272+E$40    =G271      =I272+1      =IF(C271=$A$60,"STOP","")
=B272+E$42+E$42    =D272+E$40+E$40    =D272      =B272+E$42    =G272+2      =E273+E$40    =G272      =I273+1      =IF(C272=$A$60,"STOP","")
=B273+E$42+E$42    =D273+E$40+E$40    =D273      =B273+E$42    =G273+2      =E274+E$40    =G273      =I274+1      =IF(C273=$A$60,"STOP","")
=B274+E$42+E$42    =D274+E$40+E$40    =D274      =B274+E$42    =G274+2      =E275+E$40    =G274      =I275+1      =IF(C274=$A$60,"STOP","")
=B275+E$42+E$42    =D275+E$40+E$40    =D275      =B275+E$42    =G275+2      =E276+E$40    =G275      =I276+1      =IF(C275=$A$60,"STOP","")
=B276+E$42+E$42    =D276+E$40+E$40    =D276      =B276+E$42    =G276+2      =E277+E$40    =G276      =I277+1      =IF(C276=$A$60,"STOP","")
=B277+E$42+E$42    =D277+E$40+E$40    =D277      =B277+E$42    =G277+2      =E278+E$40    =G277      =I278+1      =IF(C277=$A$60,"STOP","")
=B278+E$42+E$42    =D278+E$40+E$40    =D278      =B278+E$42    =G278+2      =E279+E$40    =G278      =I279+1      =IF(C278=$A$60,"STOP","")
=B279+E$42+E$42    =D279+E$40+E$40    =D279      =B279+E$42    =G279+2      =E280+E$40    =G279      =I280+1      =IF(C279=$A$60,"STOP","")
=B280+E$42+E$42    =D280+E$40+E$40    =D280      =B280+E$42    =G280+2      =E281+E$40    =G280      =I281+1      =IF(C280=$A$60,"STOP","")
=B281+E$42+E$42    =D281+E$40+E$40    =D281      =B281+E$42    =G281+2      =E282+E$40    =G281      =I282+1      =IF(C281=$A$60,"STOP","")
=B282+E$42+E$42    =D282+E$40+E$40    =D282      =B282+E$42    =G282+2      =E283+E$40    =G282      =I283+1      =IF(C282=$A$60,"STOP","")
=B283+E$42+E$42    =D283+E$40+E$40    =D283      =B283+E$42    =G283+2      =E284+E$40    =G283      =I284+1      =IF(C283=$A$60,"STOP","")
=B284+E$42+E$42    =D284+E$40+E$40    =D284      =B284+E$42    =G284+2      =E285+E$40    =G284      =I285+1      =IF(C284=$A$60,"STOP","")
=B285+E$42+E$42    =D285+E$40+E$40    =D285      =B285+E$42    =G285+2      =E286+E$40    =G285      =I286+1      =IF(C285=$A$60,"STOP","")
=B286+E$42+E$42    =D286+E$40+E$40    =D286      =B286+E$42    =G286+2      =E287+E$40    =G286      =I287+1      =IF(C286=$A$60,"STOP","")
=B287+E$42+E$42    =D287+E$40+E$40    =D287      =B287+E$42    =G287+2      =E288+E$40    =G287      =I288+1      =IF(C287=$A$60,"STOP","")
=B288+E$42+E$42    =D288+E$40+E$40    =D288      =B288+E$42    =G288+2      =E289+E$40    =G288      =I289+1      =IF(C288=$A$60,"STOP","")
=B289+E$42+E$42    =D289+E$40+E$40    =D289      =B289+E$42    =G289+2      =E290+E$40    =G289      =I290+1      =IF(C289=$A$60,"STOP","")
=B290+E$42+E$42    =D290+E$40+E$40    =D290      =B290+E$42    =G290+2      =E291+E$40    =G290      =I291+1      =IF(C290=$A$60,"STOP","")
=B291+E$42+E$42    =D291+E$40+E$40    =D291      =B291+E$42    =G291+2      =E292+E$40    =G291      =I292+1      =IF(C291=$A$60,"STOP","")
=B292+E$42+E$42    =D292+E$40+E$40    =D292      =B292+E$42    =G292+2      =E293+E$40    =G292      =I293+1      =IF(C292=$A$60,"STOP","")
=B293+E$42+E$42    =D293+E$40+E$40    =D293      =B293+E$42    =G293+2      =E294+E$40    =G293      =I294+1      =IF(C293=$A$60,"STOP","")
=B294+E$42+E$42    =D294+E$40+E$40    =D294      =B294+E$42    =G294+2      =E295+E$40    =G294      =I295+1      =IF(C294=$A$60,"STOP","")
=B295+E$42+E$42    =D295+E$40+E$40    =D295      =B295+E$42    =G295+2      =E296+E$40    =G295      =I296+1      =IF(C295=$A$60,"STOP","")
=B296+E$42+E$42    =D296+E$40+E$40    =D296      =B296+E$42    =G296+2      =E297+E$40    =G296      =I297+1      =IF(C296=$A$60,"STOP","")
=B297+E$42+E$42    =D297+E$40+E$40    =D297      =B297+E$42    =G297+2      =E298+E$40    =G297      =I298+1      =IF(C297=$A$60,"STOP","")
=B298+E$42+E$42    =D298+E$40+E$40    =D298      =B298+E$42    =G298+2      =E299+E$40    =G298      =I299+1      =IF(C298=$A$60,"STOP","")
=B299+E$42+E$42    =D299+E$40+E$40    =D299      =B299+E$42    =G299+2      =E300+E$40    =G299      =I300+1      =IF(C299=$A$60,"STOP","")
=B300+E$42+E$42    =D300+E$40+E$40    =D300      =B300+E$42    =G300+2      =E301+E$40    =G300      =I301+1      =IF(C300=$A$60,"STOP","")
=B301+E$42+E$42    =D301+E$40+E$40    =D301      =B301+E$42    =G301+2      =E302+E$40    =G301      =I302+1      =IF(C301=$A$60,"STOP","")
=B302+E$42+E$42    =D302+E$40+E$40    =D302      =B302+E$42    =G302+2      =E303+E$40    =G302      =I303+1      =IF(C302=$A$60,"STOP","")
=B303+E$42+E$42    =D303+E$40+E$40    =D303      =B303+E$42    =G303+2      =E304+E$40    =G303      =I304+1      =IF(C303=$A$60,"STOP","")
=B304+E$42+E$42    =D304+E$40+E$40    =D304      =B304+E$42    =G304+2      =E305+E$40    =G304      =I305+1      =IF(C304=$A$60,"STOP","")
=B305+E$42+E$42    =D305+E$40+E$40    =D305      =B305+E$42    =G305+2      =E306+E$40    =G305      =I306+1      =IF(C305=$A$60,"STOP","")
=B306+E$42+E$42    =D306+E$40+E$40    =D306      =B306+E$42    =G306+2      =E307+E$40    =G306      =I307+1      =IF(C306=$A$60,"STOP","")
=B307+E$42+E$42    =D307+E$40+E$40    =D307      =B307+E$42    =G307+2      =E308+E$40    =G307      =I308+1      =IF(C307=$A$60,"STOP","")
=B308+E$42+E$42    =D308+E$40+E$40    =D308      =B308+E$42    =G308+2      =E309+E$40    =G308      =I309+1      =IF(C308=$A$60,"STOP","")
=B309+E$42+E$42    =D309+E$40+E$40    =D309      =B309+E$42    =G309+2      =E310+E$40    =G309      =I310+1      =IF(C309=$A$60,"STOP","")
=B310+E$42+E$42    =D310+E$40+E$40    =D310      =B310+E$42    =G310+2      =E311+E$40    =G310      =I311+1      =IF(C310=$A$60,"STOP","")
=B311+E$42+E$42    =D311+E$40+E$40    =D311      =B311+E$42    =G311+2      =E312+E$40    =G311      =I312+1      =IF(C311=$A$60,"STOP","")
=B312+E$42+E$42    =D312+E$40+E$40    =D312      =B312+E$42    =G312+2      =E313+E$40    =G312      =I313+1      =IF(C312=$A$60,"STOP","")
=B313+E$42+E$42    =D313+E$40+E$40    =D313      =B313+E$42    =G313+2      =E314+E$40    =G313      =I314+1      =IF(C313=$A$60,"STOP","")

```

""
""STITCHRT
""

*ELEMENT TYPE=S9R5

=A251+E36	=A60	=B320+E42+E42	=E320+E40+E40	=E251+2"E36"E40	=B320+E42	=J513+3	=E320+E40	=G320-2	=I320+1	=IF(C320=\$A\$62,"STOP","")
=A320+1	=C320	=B321+E\$42+E\$42	=D321+E\$40+E\$40	=D321	=B321+E\$42	=G321+2	=E321+E\$40	=G321	=I321+1	=IF(C321=\$A\$62,"STOP","")
=A321+1	=C321	=B322+E\$42+E\$42	=D322+E\$40+E\$40	=D322	=B322+E\$42	=G322+2	=E322+E\$40	=G322	=I322+1	=IF(C322=\$A\$62,"STOP","")
=A322+1	=C322	=B323+E\$42+E\$42	=D323+E\$40+E\$40	=D323	=B323+E\$42	=G323+2	=E323+E\$40	=G323	=I323+1	=IF(C323=\$A\$62,"STOP","")
=A323+1	=C323	=B324+E\$42+E\$42	=D324+E\$40+E\$40	=D324	=B324+E\$42	=G324+2	=E324+E\$40	=G324	=I324+1	=IF(C324=\$A\$62,"STOP","")
=A324+1	=C324	=B325+E\$42+E\$42	=D325+E\$40+E\$40	=D325	=B325+E\$42	=G325+2	=E325+E\$40	=G325	=I325+1	=IF(C325=\$A\$62,"STOP","")
=A325+1	=C325	=B326+E\$42+E\$42	=D326+E\$40+E\$40	=D326	=B326+E\$42	=G326+2	=E326+E\$40	=G326	=I326+1	=IF(C326=\$A\$62,"STOP","")
=A326+1	=C326	=B327+E\$42+E\$42	=D327+E\$40+E\$40	=D327	=B327+E\$42	=G327+2	=E327+E\$40	=G327	=I327+1	=IF(C327=\$A\$62,"STOP","")
=A327+1	=C327	=B328+E\$42+E\$42	=D328+E\$40+E\$40	=D328	=B328+E\$42	=G328+2	=E328+E\$40	=G328	=I328+1	=IF(C328=\$A\$62,"STOP","")
=A328+1	=C328	=B329+E\$42+E\$42	=D329+E\$40+E\$40	=D329	=B329+E\$42	=G329+2	=E329+E\$40	=G329	=I329+1	=IF(C329=\$A\$62,"STOP","")
=A329+1	=C329	=B330+E\$42+E\$42	=D330+E\$40+E\$40	=D330	=B330+E\$42	=G330+2	=E330+E\$40	=G330	=I330+1	=IF(C330=\$A\$62,"STOP","")
=A330+1	=C330	=B331+E\$42+E\$42	=D331+E\$40+E\$40	=D331	=B331+E\$42	=G331+2	=E331+E\$40	=G331	=I331+1	=IF(C331=\$A\$62,"STOP","")
=A331+1	=C331	=B332+E\$42+E\$42	=D332+E\$40+E\$40	=D332	=B332+E\$42	=G332+2	=E332+E\$40	=G332	=I332+1	=IF(C332=\$A\$62,"STOP","")
=A332+1	=C332	=B333+E\$42+E\$42	=D333+E\$40+E\$40	=D333	=B333+E\$42	=G333+2	=E333+E\$40	=G333	=I333+1	=IF(C333=\$A\$62,"STOP","")
=A333+1	=C333	=B334+E\$42+E\$42	=D334+E\$40+E\$40	=D334	=B334+E\$42	=G334+2	=E334+E\$40	=G334	=I334+1	=IF(C334=\$A\$62,"STOP","")
=A334+1	=C334	=B335+E\$42+E\$42	=D335+E\$40+E\$40	=D335	=B335+E\$42	=G335+2	=E335+E\$40	=G335	=I335+1	=IF(C335=\$A\$62,"STOP","")
=A335+1	=C335	=B336+E\$42+E\$42	=D336+E\$40+E\$40	=D336	=B336+E\$42	=G336+2	=E336+E\$40	=G336	=I336+1	=IF(C336=\$A\$62,"STOP","")
=A336+1	=C336	=B337+E\$42+E\$42	=D337+E\$40+E\$40	=D337	=B337+E\$42	=G337+2	=E337+E\$40	=G337	=I337+1	=IF(C337=\$A\$62,"STOP","")
=A337+1	=C337	=B338+E\$42+E\$42	=D338+E\$40+E\$40	=D338	=B338+E\$42	=G338+2	=E338+E\$40	=G338	=I338+1	=IF(C338=\$A\$62,"STOP","")
=A338+1	=C338	=B339+E\$42+E\$42	=D339+E\$40+E\$40	=D339	=B339+E\$42	=G339+2	=E339+E\$40	=G339	=I339+1	=IF(C339=\$A\$62,"STOP","")
=A339+1	=C339	=B340+E\$42+E\$42	=D340+E\$40+E\$40	=D340	=B340+E\$42	=G340+2	=E340+E\$40	=G340	=I340+1	=IF(C340=\$A\$62,"STOP","")
=A340+1	=C340	=B341+E\$42+E\$42	=D341+E\$40+E\$40	=D341	=B341+E\$42	=G341+2	=E341+E\$40	=G341	=I341+1	=IF(C341=\$A\$62,"STOP","")
=A341+1	=C341	=B342+E\$42+E\$42	=D342+E\$40+E\$40	=D342	=B342+E\$42	=G342+2	=E342+E\$40	=G342	=I342+1	=IF(C342=\$A\$62,"STOP","")
=A342+1	=C342	=B343+E\$42+E\$42	=D343+E\$40+E\$40	=D343	=B343+E\$42	=G343+2	=E343+E\$40	=G343	=I343+1	=IF(C343=\$A\$62,"STOP","")
=A343+1	=C343	=B344+E\$42+E\$42	=D344+E\$40+E\$40	=D344	=B344+E\$42	=G344+2	=E344+E\$40	=G344	=I344+1	=IF(C344=\$A\$62,"STOP","")
=A344+1	=C344	=B345+E\$42+E\$42	=D345+E\$40+E\$40	=D345	=B345+E\$42	=G345+2	=E345+E\$40	=G345	=I345+1	=IF(C345=\$A\$62,"STOP","")
=A345+1	=C345	=B346+E\$42+E\$42	=D346+E\$40+E\$40	=D346	=B346+E\$42	=G346+2	=E346+E\$40	=G346	=I346+1	=IF(C346=\$A\$62,"STOP","")
=A346+1	=C346	=B347+E\$42+E\$42	=D347+E\$40+E\$40	=D347	=B347+E\$42	=G347+2	=E347+E\$40	=G347	=I347+1	=IF(C347=\$A\$62,"STOP","")
=A347+1	=C347	=B348+E\$42+E\$42	=D348+E\$40+E\$40	=D348	=B348+E\$42	=G348+2	=E348+E\$40	=G348	=I348+1	=IF(C348=\$A\$62,"STOP","")
=A348+1	=C348	=B349+E\$42+E\$42	=D349+E\$40+E\$40	=D349	=B349+E\$42	=G349+2	=E349+E\$40	=G349	=I349+1	=IF(C349=\$A\$62,"STOP","")
=A349+1	=C349	=B350+E\$42+E\$42	=D350+E\$40+E\$40	=D350	=B350+E\$42	=G350+2	=E350+E\$40	=G350	=I350+1	=IF(C350=\$A\$62,"STOP","")
=A350+1	=C350	=B351+E\$42+E\$42	=D351+E\$40+E\$40	=D351	=B351+E\$42	=G351+2	=E351+E\$40	=G351	=I351+1	=IF(C351=\$A\$62,"STOP","")
=A351+1	=C351	=B352+E\$42+E\$42	=D352+E\$40+E\$40	=D352	=B352+E\$42	=G352+2	=E352+E\$40	=G352	=I352+1	=IF(C352=\$A\$62,"STOP","")
=A352+1	=C352	=B353+E\$42+E\$42	=D353+E\$40+E\$40	=D353	=B353+E\$42	=G353+2	=E353+E\$40	=G353	=I353+1	=IF(C353=\$A\$62,"STOP","")
=A353+1	=C353	=B354+E\$42+E\$42	=D354+E\$40+E\$40	=D354	=B354+E\$42	=G354+2	=E354+E\$40	=G354	=I354+1	=IF(C354=\$A\$62,"STOP","")
=A354+1	=C354	=B355+E\$42+E\$42	=D355+E\$40+E\$40	=D355	=B355+E\$42	=G355+2	=E355+E\$40	=G355	=I355+1	=IF(C355=\$A\$62,"STOP","")
=A355+1	=C355	=B356+E\$42+E\$42	=D356+E\$40+E\$40	=D356	=B356+E\$42	=G356+2	=E356+E\$40	=G356	=I356+1	=IF(C356=\$A\$62,"STOP","")
=A356+1	=C356	=B357+E\$42+E\$42	=D357+E\$40+E\$40	=D357	=B357+E\$42	=G357+2	=E357+E\$40	=G357	=I357+1	=IF(C357=\$A\$62,"STOP","")
=A357+1	=C357	=B358+E\$42+E\$42	=D358+E\$40+E\$40	=D358	=B358+E\$42	=G358+2	=E358+E\$40	=G358	=I358+1	=IF(C358=\$A\$62,"STOP","")
=A358+1	=C358	=B359+E\$42+E\$42	=D359+E\$40+E\$40	=D359	=B359+E\$42	=G359+2	=E359+E\$40	=G359	=I359+1	=IF(C359=\$A\$62,"STOP","")
=A359+1	=C359	=B360+E\$42+E\$42	=D360+E\$40+E\$40	=D360	=B360+E\$42	=G360+2	=E360+E\$40	=G360	=I360+1	=IF(C360=\$A\$62,"STOP","")
=A360+1	=C360	=B361+E\$42+E\$42	=D361+E\$40+E\$40	=D361	=B361+E\$42	=G361+2	=E361+E\$40	=G361	=I361+1	=IF(C361=\$A\$62,"STOP","")
=A361+1	=C361	=B362+E\$42+E\$42	=D362+E\$40+E\$40	=D362	=B362+E\$42	=G362+2	=E362+E\$40	=G362	=I362+1	=IF(C362=\$A\$62,"STOP","")
=A362+1	=C362	=B363+E\$42+E\$42	=D363+E\$40+E\$40	=D363	=B363+E\$42	=G363+2	=E363+E\$40	=G363	=I363+1	=IF(C363=\$A\$62,"STOP","")
=A363+1	=C363	=B364+E\$42+E\$42	=D364+E\$40+E\$40	=D364	=B364+E\$42	=G364+2	=E364+E\$40	=G364	=I364+1	=IF(C364=\$A\$62,"STOP","")
=A364+1	=C364	=B365+E\$42+E\$42	=D365+E\$40+E\$40	=D365	=B365+E\$42	=G365+2	=E365+E\$40	=G365	=I365+1	=IF(C365=\$A\$62,"STOP","")
=A365+1	=C365	=B366+E\$42+E\$42	=D366+E\$40+E\$40	=D366	=B366+E\$42	=G366+2	=E366+E\$40	=G366	=I366+1	=IF(C366=\$A\$62,"STOP","")
=A366+1	=C366	=B367+E\$42+E\$42	=D367+E\$40+E\$40	=D367	=B367+E\$42	=G367+2	=E367+E\$40	=G367	=I367+1	=IF(C367=\$A\$62,"STOP","")
=A367+1	=C367	=B368+E\$42+E\$42	=D368+E\$40+E\$40	=D368	=B368+E\$42	=G368+2	=E368+E\$40	=G368	=I368+1	=IF(C368=\$A\$62,"STOP","")
=A368+1	=C368	=B369+E\$42+E\$42	=D369+E\$40+E\$40	=D369	=B369+E\$42	=G369+2	=E369+E\$40	=G369	=I369+1	=IF(C369=\$A\$62,"STOP","")
=A369+1	=C369	=B370+E\$42+E\$42	=D370+E\$40+E\$40	=D370	=B370+E\$42	=G370+2	=E370+E\$40	=G370	=I370+1	=IF(C370=\$A\$62,"STOP","")
=A370+1	=C370	=B371+E\$42+E\$42	=D371+E\$40+E\$40	=D371	=B371+E\$42	=G371+2	=E371+E\$40	=G371	=I371+1	=IF(C371=\$A\$62,"STOP","")
=A371+1	=C371	=B372+E\$42+E\$42	=D372+E\$40+E\$40	=D372	=B372+E\$42	=G372+2	=E372+E\$40	=G372	=I372+1	=IF(C372=\$A\$62,"STOP","")
=A372+1	=C372	=B373+E\$42+E\$42	=D373+E\$40+E\$40	=D373	=B373+E\$42	=G373+2	=E373+E\$40	=G373	=I373+1	=IF(C373=\$A\$62,"STOP","")
=A373+1	=C373	=B374+E\$42+E\$42	=D374+E\$40+E\$40	=D374	=B374+E\$42	=G374+2	=E374+E\$40	=G374	=I374+1	=IF(C374=\$A\$62,"STOP","")
=A374+1	=C374	=B375+E\$42+E\$42	=D375+E\$40+E\$40	=D375	=B375+E\$42	=G375+2	=E375+E\$40	=G375	=I375+1	=IF(C375=\$A\$62,"STOP","")
=A375+1	=C375	=B376+E\$42+E\$42	=D376+E\$40+E\$40	=D376	=B376+E\$42	=G376+2	=E376+E\$40	=G376	=I376+1	=IF(C376=\$A\$62,"STOP","")
=A376+1	=C376	=B377+E\$42+E\$42	=D377+E\$40+E\$40	=D377	=B377+E\$42	=G377+2	=E377+E\$40	=G377	=I377+1	=IF(C377=\$A\$62,"STOP","")
=A377+1	=C377	=B378+E\$42+E\$42	=D378+E\$40+E\$40	=D378	=B378+E\$42	=G378+2	=E378+E\$40	=G378	=I378+1	=IF(C378=\$A\$62,"STOP","")
=A378+1	=C378	=B379+E\$42+E\$42	=D379+E\$40+E\$40	=D379	=B379+E\$42	=G379+2	=E379+E\$40	=G379	=I379+1	=IF(C379=\$A\$62,"STOP","")
=A379+1	=C379	=B380+E\$42+E\$42	=D380+E\$40+E\$40	=D380	=B380+E\$42	=G380+2	=E380+E\$40	=G380	=I380+1	=IF(C380=\$A\$62,"STOP","")
=A380+1	=C380	=B381+E\$42+E\$42	=D381+E\$40+E\$40	=D381	=B381+E\$42	=G381+2	=E381+E\$40	=G381	=I381+1	=IF(C381=\$A\$62,"STOP","")
=A381+1	=C381	=B382+E\$42+E\$42	=D382+E\$40+E\$40	=D382	=B382+E\$42	=G382+2	=E382+E\$40	=G382	=I382+1	=IF(C382=\$A\$62,"STOP","")
*ELSET	ELSET=STITCHRT	GENERATE								
=A320	=A382	1								

```

**
**SECTION PROPERTIES
**
*SHELL SECTION          ELSET=FLANGELT          MATERIAL=RIGID
=C23
*SHELL SECTION          ELSET=WEBLT            MATERIAL=RIGID
=C24
*SHELL SECTION          ELSET=STITCHLT          MATERIAL=RIGID
=C24
*SHELL SECTION          ELSET=FLANGERT          MATERIAL=RIGID
=C23
*SHELL SECTION          ELSET=WEBRT            MATERIAL=RIGID
=C24
*SHELL SECTION          ELSET=STITCHRT          MATERIAL=RIGID
=C24
*SHELL SECTION          ELSET=FLANGEMID          MATERIAL=STEEL
=C23
*SHELL SECTION          ELSET=WEBMID            MATERIAL=STEEL
=C24
*SHELL SECTION          ELSET=STITCHMID          MATERIAL=STEEL
=C24
**
**MATERIAL PROPERTIES
**
*MATERIAL                NAME=RIGID
*ELASTIC
=F24                    0.3
*MATERIAL                NAME=STEEL
*ELASTIC
=F19                    0.3
*PLASTIC
=F20                    0
=A416+1.345             0.00922948
=A416+25                0.0557238
=A416+30                0.090034
**
**BOUNDARY CONDITIONS
**
*BOUNDARY
FLPINLT                1
FLPINLT                2
FLPINLT                3
FLPINRT                2
FLPINRT                3
WEBPINLT               2
WEBPINRT               2
MIDPINLT               2
MIDPINRT               2
**
**ANALYSIS                INPUT FILE
**
*STEP
*BUCKLE
10                      20          80
*LOAD
LTLOAD                 3          1
RTLOAD                 3          1
*RESTART               WRITE        FREQUENCY=1
*NODE FILE             LAST MODE=1  GLOBAL=YES
U
*ELPRINT               FREQUENCY=0
*END STEP
**
**ANALYSIS                MATERIAL PROPERTIES
**
*IMPERFECTION           FILE=          =C3          STEP=1
1                       =C19/1000
*STEP                   NLGEOM
*STATIC                 RIKS
0.001                   1
*LOAD
LTLOAD                 3          1
RTLOAD                 3          1
*RESTART               WRITE        FREQUENCY=1
*END STEP

```

Appendix A3: Development Spreadsheet

Filename var2_1.25

*Heading
alter tw while bf/d=2.0
RIGID ENDS
S9R5 ELEMENTS
APPROXIMATELY 1" ELEMENTS
ACTUAL WTEE DIMENSIONS
**
**INITIAL CONDITIONS
**

INPUT VARIABLES

INCHES		STEEL	
			KSI
Lb=	176.573	E=	29000
L=	529.719	Fy=	50
bf=	28	v=	0.3
d=	14		
tf=	1.523	RIGID	
tw=	1.25	E=	290000

NUMBER OF ELEMENTS IN FLANGE	10 THIS NUMBER CAN BE ALTERED
NUMBER OF ELEMENTS IN LENGTH	63.06179

FLANGE ELEMENT SIZE	2.8	midpoint	3981
LENGTH ELEMENT SIZE	2.802746		
WEB ELEMENT SIZE	1.4125		

MESH SETUP

EL FL	10	10	THIS SHOULD BE EVEN
EL Lb	63	63	
EL WB	8.91150442	9	
STITCH	1.2875	1.288	
FLANGE LENGTH INC		21	
FLANGE INC		20	
WEB LENGTH INC		19	
WEB INC		18	

INITIAL COORDINATES

*NODE

1	0	0	0
21	0	28	0
2647	176.573	0	0
2667	176.573	28	0
5293	353.146	0	0
5313	353.146	28	0
7939	529.719	0	0
7959	529.719	28	0
7960	0	14	-14.0005
7978	0	14	-1.288
10354	176.573	14	-14.0005
10372	176.573	14	-1.288
12748	353.146	14	-14.0005
12766	353.146	14	-1.288
15142	529.719	14	-14.0005
15160	529.719	14	-1.288
51000	0	14	-0.644
51378	529.719	14	-0.644

First node of stitch, start at even number,
must be greater than last assigned node

```

**
**NODE DEFINITION
**
*NGEN          NSET=BFLANGELT
              1          2647      21
*NGEN          NSET=TFLANGELT
              21          2667      21
*NGEN          NSET=BFLANGEMID
              2647          5293      21
*NGEN          NSET=TFLANGEMID
              2667          5313      21
*NGEN          NSET=BFLANGERT
              5293          7939      21
*NGEN          NSET=TFLANGERT
              5313          7959      21
*NGEN          NSET=BWEBLT
              7960          10354      19
*NGEN          NSET=TWEBLT
              7978          10372      19
*NGEN          NSET=BWEBMID
              10354          12748      19
*NGEN          NSET=TWEBMID
              10372          12766      19
*NGEN          NSET=BWEBRT
              12748          15142      19
*NGEN          NSET=TWEBRT
              12766          15160      19
*NGEN          NSET=STITCH
              51000          51378      1

**
**FILL NODES
**

*NFILL          NSET=FLANGELT
BFLANGELT      TFLANGELT          20
*NFILL          NSET=FLANGEMID
BFLANGEMID      TFLANGEMID          20
*NFILL          NSET=FLANGERT
BFLANGERT      TFLANGERT          20
*NFILL          NSET=WEBLT
BWEBLT          TWEBLT          18
*NFILL          NSET=WEBMID
BWEBMID          TWEBMID          18
*NFILL          NSET=WEBRT
BWEBRT          TWEBRT          18

**
**CONSTRAINT SETS
**

*NSET          NSET=FLPINLT      GENERATE
              1          21          1
*NSET          NSET=FLPINRT      GENERATE
              7939          7959          1
*NSET          NSET=WEBPINLT      GENERATE
              7960          7978          1
*NSET          NSET=WEBPINRT      GENERATE
              15142          15160          1
*NSET          NSET=MIDPINLT      GENERATE
              10354          10372          1
*NSET          NSET=MIDPINRT      GENERATE
              12748          12766          1
*NSET          NSET=LTLOAD      GENERATE
              2647          2667          1
*NSET          NSET=RTLOAD      GENERATE
              5293          5313          1

```

```

**
**FLANGELT
**
*ELEMENT      MASTER ELEMENT
               TYPE=S9R5
1              1      43      45      3      22      44      24      2      23
               GENERATE ELEMENT SET
*ELGEN        ELSET=FLANGELT
1              63      42      1      10      2      189
**
**FLANGEMID
**
*ELEMENT      TYPE=S9R5
64             2647      2689      2691      2649      2668      2690      2670      2648      2669
*ELGEN        ELSET=FLANGEMID
64             63      42      1      10      2      189
**
**FLANGERT
**
*ELEMENT      TYPE=S9R5
127            5293      5335      5337      5295      5314      5336      5316      5294      5315
*ELGEN        ELSET=FLANGERT
127            63      42      1      10      2      189
**
**WEBLT
**
*ELEMENT      TYPE=S9R5
1891           7960      7998      8000      7962      7979      7999      7981      7961      7980
*ELGEN        ELSET=WEBLT
1891           63      38      1      9      2      189
**
**WEBMID
**
*ELEMENT      TYPE=S9R5
1954           10354      10392      10394      10356      10373      10393      10375      10355      10374
*ELGEN        ELSET=WEBMID
1954           63      38      1      9      2      189
**
**WEBRT
**
*ELEMENT      TYPE=S9R5
2017           12748      12786      12788      12750      12767      12787      12769      12749      12768
*ELGEN        ELSET=WEBRT
2017           63      38      1      9      2      189

```

```

**
**STITCHLT
**
*ELEMENT      TYPE=S9R5
3592          7978      8016      53      11      7997      51002      32      51000      51001
3593          8016      8054      95      53      8035      51004      74      51002      51003
3594          8054      8092      137     95      8073      51006      116     51004      51005
3595          8092      8130      179    137     8111      51008      158     51006      51007
3596          8130      8168      221    179     8149      51010      200     51008      51009
3597          8168      8206      263    221     8187      51012      242     51010      51011
3598          8206      8244      305    263     8225      51014      284     51012      51013
3599          8244      8282      347    305     8263      51016      326     51014      51015
3600          8282      8320      389    347     8301      51018      368     51016      51017
3601          8320      8358      431    389     8339      51020      410     51018      51019
3602          8358      8396      473    431     8377      51022      452     51020      51021
3603          8396      8434      515    473     8415      51024      494     51022      51023
3604          8434      8472      557    515     8453      51026      536     51024      51025
3605          8472      8510      599    557     8491      51028      578     51026      51027
3606          8510      8548      641    599     8529      51030      620     51028      51029
3607          8548      8586      683    641     8567      51032      662     51030      51031
3608          8586      8624      725    683     8605      51034      704     51032      51033
3609          8624      8662      767    725     8643      51036      746     51034      51035
3610          8662      8700      809    767     8681      51038      788     51036      51037
3611          8700      8738      851    809     8719      51040      830     51038      51039
3612          8738      8776      893    851     8757      51042      872     51040      51041
3613          8776      8814      935    893     8795      51044      914     51042      51043
3614          8814      8852      977    935     8833      51046      956     51044      51045
3615          8852      8890      1019   977     8871      51048      998     51046      51047
3616          8890      8928      1061   1019    8909      51050      1040    51048      51049
3617          8928      8966      1103   1061    8947      51052      1082    51050      51051
3618          8966      9004      1145   1103    8985      51054      1124    51052      51053
3619          9004      9042      1187   1145    9023      51056      1166    51054      51055
3620          9042      9080      1229   1187    9061      51058      1208    51056      51057
3621          9080      9118      1271   1229    9099      51060      1250    51058      51059
3622          9118      9156      1313   1271    9137      51062      1292    51060      51061
3623          9156      9194      1355   1313    9175      51064      1334    51062      51063
3624          9194      9232      1397   1355    9213      51066      1376    51064      51065
3625          9232      9270      1439   1397    9251      51068      1418    51066      51067
3626          9270      9308      1481   1439    9289      51070      1460    51068      51069
3627          9308      9346      1523   1481    9327      51072      1502    51070      51071
3628          9346      9384      1565   1523    9365      51074      1544    51072      51073
3629          9384      9422      1607   1565    9403      51076      1586    51074      51075
3630          9422      9460      1649   1607    9441      51078      1628    51076      51077
3631          9460      9498      1691   1649    9479      51080      1670    51078      51079
3632          9498      9536      1733   1691    9517      51082      1712    51080      51081
3633          9536      9574      1775   1733    9555      51084      1754    51082      51083
3634          9574      9612      1817   1775    9593      51086      1796    51084      51085
3635          9612      9650      1859   1817    9631      51088      1838    51086      51087
3636          9650      9688      1901   1859    9669      51090      1880    51088      51089
3637          9688      9726      1943   1901    9707      51092      1922    51090      51091
3638          9726      9764      1985   1943    9745      51094      1964    51092      51093
3639          9764      9802      2027   1985    9783      51096      2006    51094      51095
3640          9802      9840      2069   2027    9821      51098      2048    51096      51097
3641          9840      9878      2111   2069    9859      51100      2090    51098      51099
3642          9878      9916      2153   2111    9897      51102      2132    51100      51101
3643          9916      9954      2195   2153    9935      51104      2174    51102      51103
3644          9954      9992      2237   2195    9973      51106      2216    51104      51105
3645          9992      10030     2279   2237    10011     51108      2258    51106      51107
3646          10030     10068     2321   2279    10049     51110      2300    51108      51109
3647          10068     10106     2363   2321    10087     51112      2342    51110      51111
3648          10106     10144     2405   2363    10125     51114      2384    51112      51113
3649          10144     10182     2447   2405    10163     51116      2426    51114      51115
3650          10182     10220     2489   2447    10201     51118      2468    51116      51117
3651          10220     10258     2531   2489    10239     51120      2510    51118      51119
3652          10258     10296     2573   2531    10277     51122      2552    51120      51121
3653          10296     10334     2615   2573    10315     51124      2594    51122      51123
3654          10334     10372     2657   2615    10353     51126      2636    51124      51125 STOP
*ELSET      ELSET=STITCHLT  GENERATE
3592          3654      1

```



```

**
**STITCHMID
**
*ELEMENT      TYPE=S9R5
3655          10372  10410  2699  2657  10391  51128  2678  51126  51127
3656          10410  10448  2741  2699  10429  51130  2720  51128  51129
3657          10448  10486  2783  2741  10467  51132  2762  51130  51131
3658          10486  10524  2825  2783  10505  51134  2804  51132  51133
3659          10524  10562  2867  2825  10543  51136  2846  51134  51135
3660          10562  10600  2909  2867  10581  51138  2888  51136  51137
3661          10600  10638  2951  2909  10619  51140  2930  51138  51139
3662          10638  10676  2993  2951  10657  51142  2972  51140  51141
3663          10676  10714  3035  2993  10695  51144  3014  51142  51143
3664          10714  10752  3077  3035  10733  51146  3056  51144  51145
3665          10752  10790  3119  3077  10771  51148  3098  51146  51147
3666          10790  10828  3161  3119  10809  51150  3140  51148  51149
3667          10828  10866  3203  3161  10847  51152  3182  51150  51151
3668          10866  10904  3245  3203  10885  51154  3224  51152  51153
3669          10904  10942  3287  3245  10923  51156  3266  51154  51155
3670          10942  10980  3329  3287  10961  51158  3308  51156  51157
3671          10980  11018  3371  3329  10999  51160  3350  51158  51159
3672          11018  11056  3413  3371  11037  51162  3392  51160  51161
3673          11056  11094  3455  3413  11075  51164  3434  51162  51163
3674          11094  11132  3497  3455  11113  51166  3476  51164  51165
3675          11132  11170  3539  3497  11151  51168  3518  51166  51167
3676          11170  11208  3581  3539  11189  51170  3560  51168  51169
3677          11208  11246  3623  3581  11227  51172  3602  51170  51171
3678          11246  11284  3665  3623  11265  51174  3644  51172  51173
3679          11284  11322  3707  3665  11303  51176  3686  51174  51175
3680          11322  11360  3749  3707  11341  51178  3728  51176  51177
3681          11360  11398  3791  3749  11379  51180  3770  51178  51179
3682          11398  11436  3833  3791  11417  51182  3812  51180  51181
3683          11436  11474  3875  3833  11455  51184  3854  51182  51183
3684          11474  11512  3917  3875  11493  51186  3896  51184  51185
3685          11512  11550  3959  3917  11531  51188  3938  51186  51187
3686          11550  11588  4001  3959  11569  51190  3980  51188  51189
3687          11588  11626  4043  4001  11607  51192  4022  51190  51191
3688          11626  11664  4085  4043  11645  51194  4064  51192  51193
3689          11664  11702  4127  4085  11683  51196  4106  51194  51195
3690          11702  11740  4169  4127  11721  51198  4148  51196  51197
3691          11740  11778  4211  4169  11759  51200  4190  51198  51199
3692          11778  11816  4253  4211  11797  51202  4232  51200  51201
3693          11816  11854  4295  4253  11835  51204  4274  51202  51203
3694          11854  11892  4337  4295  11873  51206  4316  51204  51205
3695          11892  11930  4379  4337  11911  51208  4358  51206  51207
3696          11930  11968  4421  4379  11949  51210  4400  51208  51209
3697          11968  12006  4463  4421  11987  51212  4442  51210  51211
3698          12006  12044  4505  4463  12025  51214  4484  51212  51213
3699          12044  12082  4547  4505  12063  51216  4526  51214  51215
3700          12082  12120  4589  4547  12101  51218  4568  51216  51217
3701          12120  12158  4631  4589  12139  51220  4610  51218  51219
3702          12158  12196  4673  4631  12177  51222  4652  51220  51221
3703          12196  12234  4715  4673  12215  51224  4694  51222  51223
3704          12234  12272  4757  4715  12253  51226  4736  51224  51225
3705          12272  12310  4799  4757  12291  51228  4778  51226  51227
3706          12310  12348  4841  4799  12329  51230  4820  51228  51229
3707          12348  12386  4883  4841  12367  51232  4862  51230  51231
3708          12386  12424  4925  4883  12405  51234  4904  51232  51233
3709          12424  12462  4967  4925  12443  51236  4946  51234  51235
3710          12462  12500  5009  4967  12481  51238  4988  51236  51237
3711          12500  12538  5051  5009  12519  51240  5030  51238  51239
3712          12538  12576  5093  5051  12557  51242  5072  51240  51241
3713          12576  12614  5135  5093  12595  51244  5114  51242  51243
3714          12614  12652  5177  5135  12633  51246  5156  51244  51245
3715          12652  12690  5219  5177  12671  51248  5198  51246  51247
3716          12690  12728  5261  5219  12709  51250  5240  51248  51249
3717          12728  12766  5303  5261  12747  51252  5282  51250  51251 STOP
*ELSET      ELSET=STITCHMID GENERATE
3655          3717      1

```

```

**
**STITCHRT
**
*ELEMENT      TYPE=S9R5
3718          12766  12804  5345  5303  12785  51254  5324  51252  51253
3719          12804  12842  5387  5345  12823  51256  5366  51254  51255
3720          12842  12880  5429  5387  12861  51258  5408  51256  51257
3721          12880  12918  5471  5429  12899  51260  5450  51258  51259
3722          12918  12956  5513  5471  12937  51262  5492  51260  51261
3723          12956  12994  5555  5513  12975  51264  5534  51262  51263
3724          12994  13032  5597  5555  13013  51266  5576  51264  51265
3725          13032  13070  5639  5597  13051  51268  5618  51266  51267
3726          13070  13108  5681  5639  13089  51270  5660  51268  51269
3727          13108  13146  5723  5681  13127  51272  5702  51270  51271
3728          13146  13184  5765  5723  13165  51274  5744  51272  51273
3729          13184  13222  5807  5765  13203  51276  5786  51274  51275
3730          13222  13260  5849  5807  13241  51278  5828  51276  51277
3731          13260  13298  5891  5849  13279  51280  5870  51278  51279
3732          13298  13336  5933  5891  13317  51282  5912  51280  51281
3733          13336  13374  5975  5933  13355  51284  5954  51282  51283
3734          13374  13412  6017  5975  13393  51286  5996  51284  51285
3735          13412  13450  6059  6017  13431  51288  6038  51286  51287
3736          13450  13488  6101  6059  13469  51290  6080  51288  51289
3737          13488  13526  6143  6101  13507  51292  6122  51290  51291
3738          13526  13564  6185  6143  13545  51294  6164  51292  51293
3739          13564  13602  6227  6185  13583  51296  6206  51294  51295
3740          13602  13640  6269  6227  13621  51298  6248  51296  51297
3741          13640  13678  6311  6269  13659  51300  6290  51298  51299
3742          13678  13716  6353  6311  13697  51302  6332  51300  51301
3743          13716  13754  6395  6353  13735  51304  6374  51302  51303
3744          13754  13792  6437  6395  13773  51306  6416  51304  51305
3745          13792  13830  6479  6437  13811  51308  6458  51306  51307
3746          13830  13868  6521  6479  13849  51310  6500  51308  51309
3747          13868  13906  6563  6521  13887  51312  6542  51310  51311
3748          13906  13944  6605  6563  13925  51314  6584  51312  51313
3749          13944  13982  6647  6605  13963  51316  6626  51314  51315
3750          13982  14020  6689  6647  14001  51318  6668  51316  51317
3751          14020  14058  6731  6689  14039  51320  6710  51318  51319
3752          14058  14096  6773  6731  14077  51322  6752  51320  51321
3753          14096  14134  6815  6773  14115  51324  6794  51322  51323
3754          14134  14172  6857  6815  14153  51326  6836  51324  51325
3755          14172  14210  6899  6857  14191  51328  6878  51326  51327
3756          14210  14248  6941  6899  14229  51330  6920  51328  51329
3757          14248  14286  6983  6941  14267  51332  6962  51330  51331
3758          14286  14324  7025  6983  14305  51334  7004  51332  51333
3759          14324  14362  7067  7025  14343  51336  7046  51334  51335
3760          14362  14400  7109  7067  14381  51338  7088  51336  51337
3761          14400  14438  7151  7109  14419  51340  7130  51338  51339
3762          14438  14476  7193  7151  14457  51342  7172  51340  51341
3763          14476  14514  7235  7193  14495  51344  7214  51342  51343
3764          14514  14552  7277  7235  14533  51346  7256  51344  51345
3765          14552  14590  7319  7277  14571  51348  7298  51346  51347
3766          14590  14628  7361  7319  14609  51350  7340  51348  51349
3767          14628  14666  7403  7361  14647  51352  7382  51350  51351
3768          14666  14704  7445  7403  14685  51354  7424  51352  51353
3769          14704  14742  7487  7445  14723  51356  7466  51354  51355
3770          14742  14780  7529  7487  14761  51358  7508  51356  51357
3771          14780  14818  7571  7529  14799  51360  7550  51358  51359
3772          14818  14856  7613  7571  14837  51362  7592  51360  51361
3773          14856  14894  7655  7613  14875  51364  7634  51362  51363
3774          14894  14932  7697  7655  14913  51366  7676  51364  51365
3775          14932  14970  7739  7697  14951  51368  7718  51366  51367
3776          14970  15008  7781  7739  14989  51370  7760  51368  51369
3777          15008  15046  7823  7781  15027  51372  7802  51370  51371
3778          15046  15084  7865  7823  15065  51374  7844  51372  51373
3779          15084  15122  7907  7865  15103  51376  7886  51374  51375
3780          15122  15160  7949  7907  15141  51378  7928  51376  51377 STOP
*ELSET      ELSET=STITCHRT  GENERATE
3718          3780          1

```

```

**
**SECTION PROPERTIES
**
*SHELL SECTION ELSET=FLANGELT MATERIAL=RIGID
1.523
*SHELL SECTION ELSET=WEBLT MATERIAL=RIGID
1.25
*SHELL SECTION ELSET=STITCHLT MATERIAL=RIGID
1.25
*SHELL SECTION ELSET=FLANGERT MATERIAL=RIGID
1.523
*SHELL SECTION ELSET=WEBRT MATERIAL=RIGID
1.25
*SHELL SECTION ELSET=STITCHRT MATERIAL=RIGID
1.25
*SHELL SECTION ELSET=FLANGEMID MATERIAL=STEEL
1.523
*SHELL SECTION ELSET=WEBMID MATERIAL=STEEL
1.25
*SHELL SECTION ELSET=STITCHMID MATERIAL=STEEL
1.25
**
**MATERIAL PROPERTIES
**
*MATERIAL NAME=RIGID
*ELASTIC
2.90E+05 0.3
*MATERIAL NAME=STEEL
*ELASTIC
2.90E+04 0.3
*PLASTIC
50 0
51.345 0.00922948
75 0.0557238
80 0.090034
**
**BOUNDARY CONDITIONS
**
*BOUNDARY
FLPINLT 1
FLPINLT 2
FLPINLT 3
FLPINRT 2
FLPINRT 3
WEBPINLT 2
WEBPINRT 2
MIDPINLT 2
MIDPINRT 2
**
**ANALYSIS INPUT FILE
**
*STEP
*BUCKLE
10 20 80
*CLOAD
LTLOAD 3 1
RTLOAD 3 1
*RESTART WRITE FREQUENCY=1
*NODE FILE LAST MODE=1 GLOBAL=YES
U
*ELPRINT FREQUENCY=0
*END STEP
**
**ANALYSIS MATERIAL PROPERTIES
**
*IMPERFECTION FILE= var2_1.25 STEP=1
1 0.176573
*STEP NLGEOM INC=50
*STATIC RIKS
0.001 1
*CLOAD
LTLOAD 3 1
RTLOAD 3 1
*RESTART WRITE FREQUENCY=1
*END STEP

```

Appendix A4: Input Files

*HEADING – SEED IMPERFECTION FILE

ALTER tw WHILE bf/d=2.0

RIGID ENDS

S9R5 ELEMENTS

APPROXIMATELY 1" ELEMENTS

ACTUAL WTEE DIMENSIONS

**

**INITIAL CONDITIONS

**

*NODE

1,0,0,0

21,0,28,0

2647,176.573,0,0

2667,176.573,28,0

5293,353.146,0,0

5313,353.146,28,0

7939,529.719,0,0

7959,529.719,28,0

7960,0,14,-14.0005

7978,0,14,-1.288

10354,176.573,14,-14.0005

10372,176.573,14,-1.288

12748,353.146,14,-14.0005

12766,353.146,14,-1.288

15142,529.719,14,-14.0005

15160,529.719,14,-1.288

51000,0,14,-0.644

51378,529.719,14,-0.644

**

**NODE DEFINITION

**

*NGEN,NSET=BFLANGELT

1,2647,21

*NGEN,NSET=TFLANGELT

21,2667,21

*NGEN,NSET=BFLANGEMID

2647,5293,21

*NGEN,NSET=TFLANGEMID

2667,5313,21

*NGEN,NSET=BFLANGERT

5293,7939,21

*NGEN,NSET=TFLANGERT

5313,7959,21

```

*NGEN,NSET=BWEBLT
7960,10354,19
*NGEN,NSET=TWEBLT
7978,10372,19
*NGEN,NSET=BWEBMID
10354,12748,19
*NGEN,NSET=TWEBMID
10372,12766,19
*NGEN,NSET=BWEBRT
12748,15142,19
*NGEN,NSET=TWEBRT
12766,15160,19
*NGEN,NSET=STITCH
51000,51378,1
**
**FILL NODES
**
*NFILL,NSET=FLANGELT
BFLANGELT,TFLANGELT,20
*NFILL,NSET=FLANGEMID
BFLANGEMID,TFLANGEMID,20
*NFILL,NSET=FLANGERT
BFLANGERT,TFLANGERT,20
*NFILL,NSET=WEBLT
BWEBLT,TWEBLT,18
*NFILL,NSET=WEBMID
BWEBMID,TWEBMID,18
*NFILL,NSET=WEBRT
BWEBRT,TWEBRT,18
**
**CONSTRAINT SETS
**
*NSET,NSET=FLPINLT,GENERATE
1,21,1
*NSET,NSET=FLPINRT,GENERATE
7939,7959,1
*NSET,NSET=WEBPINLT,GENERATE
7960,7978,1
*NSET,NSET=WEBPINRT,GENERATE
15142,15160,1
*NSET,NSET=MIDPINLT,GENERATE
10354,10372,1
*NSET,NSET=MIDPINRT,GENERATE
12748,12766,1
*NSET,NSET=LTLOAD,GENERATE
2647,2667,1

```

```

*NSET,NSET=RTLOAD,GENERATE
5293,5313,1
**
**FLANGELT
**
*ELEMENT,TYPE=S9R5
1,1,43,45,3,22,44,24,2,23
*ELGEN,ELSET=FLANGELT
1,63,42,1,10,2,189
**
**FLANGEMID
**
*ELEMENT,TYPE=S9R5
64,2647,2689,2691,2649,2668,2690,2670,2648,2669
*ELGEN,ELSET=FLANGEMID
64,63,42,1,10,2,189
**
**FLANGERT
**
*ELEMENT,TYPE=S9R5
127,5293,5335,5337,5295,5314,5336,5316,5294,5315
*ELGEN,ELSET=FLANGERT
127,63,42,1,10,2,189
**
**WEBLT
**
*ELEMENT,TYPE=S9R5
1891,7960,7998,8000,7962,7979,7999,7981,7961,7980
*ELGEN,ELSET=WEBLT
1891,63,38,1,9,2,189
**
**WEBMID
**
*ELEMENT,TYPE=S9R5
1954,10354,10392,10394,10356,10373,10393,10375,10355,10374
*ELGEN,ELSET=WEBMID
1954,63,38,1,9,2,189
**
**WEBRT
**
*ELEMENT,TYPE=S9R5
2017,12748,12786,12788,12750,12767,12787,12769,12749,12768
*ELGEN,ELSET=WEBRT
2017,63,38,1,9,2,189

```

**

**STITCHLT

**

*ELEMENT,TYPE=S9R5

3592,7978,8016,53,11,7997,51002,32,51000,51001
3593,8016,8054,95,53,8035,51004,74,51002,51003
3594,8054,8092,137,95,8073,51006,116,51004,51005
3595,8092,8130,179,137,8111,51008,158,51006,51007
3596,8130,8168,221,179,8149,51010,200,51008,51009
3597,8168,8206,263,221,8187,51012,242,51010,51011
3598,8206,8244,305,263,8225,51014,284,51012,51013
3599,8244,8282,347,305,8263,51016,326,51014,51015
3600,8282,8320,389,347,8301,51018,368,51016,51017
3601,8320,8358,431,389,8339,51020,410,51018,51019
3602,8358,8396,473,431,8377,51022,452,51020,51021
3603,8396,8434,515,473,8415,51024,494,51022,51023
3604,8434,8472,557,515,8453,51026,536,51024,51025
3605,8472,8510,599,557,8491,51028,578,51026,51027
3606,8510,8548,641,599,8529,51030,620,51028,51029
3607,8548,8586,683,641,8567,51032,662,51030,51031
3608,8586,8624,725,683,8605,51034,704,51032,51033
3609,8624,8662,767,725,8643,51036,746,51034,51035
3610,8662,8700,809,767,8681,51038,788,51036,51037
3611,8700,8738,851,809,8719,51040,830,51038,51039
3612,8738,8776,893,851,8757,51042,872,51040,51041
3613,8776,8814,935,893,8795,51044,914,51042,51043
3614,8814,8852,977,935,8833,51046,956,51044,51045
3615,8852,8890,1019,977,8871,51048,998,51046,51047
3616,8890,8928,1061,1019,8909,51050,1040,51048,51049
3617,8928,8966,1103,1061,8947,51052,1082,51050,51051
3618,8966,9004,1145,1103,8985,51054,1124,51052,51053
3619,9004,9042,1187,1145,9023,51056,1166,51054,51055
3620,9042,9080,1229,1187,9061,51058,1208,51056,51057
3621,9080,9118,1271,1229,9099,51060,1250,51058,51059
3622,9118,9156,1313,1271,9137,51062,1292,51060,51061
3623,9156,9194,1355,1313,9175,51064,1334,51062,51063
3624,9194,9232,1397,1355,9213,51066,1376,51064,51065
3625,9232,9270,1439,1397,9251,51068,1418,51066,51067
3626,9270,9308,1481,1439,9289,51070,1460,51068,51069
3627,9308,9346,1523,1481,9327,51072,1502,51070,51071
3628,9346,9384,1565,1523,9365,51074,1544,51072,51073
3629,9384,9422,1607,1565,9403,51076,1586,51074,51075
3630,9422,9460,1649,1607,9441,51078,1628,51076,51077
3631,9460,9498,1691,1649,9479,51080,1670,51078,51079

3632,9498,9536,1733,1691,9517,51082,1712,51080,51081
 3633,9536,9574,1775,1733,9555,51084,1754,51082,51083
 3634,9574,9612,1817,1775,9593,51086,1796,51084,51085
 3635,9612,9650,1859,1817,9631,51088,1838,51086,51087
 3636,9650,9688,1901,1859,9669,51090,1880,51088,51089
 3637,9688,9726,1943,1901,9707,51092,1922,51090,51091
 3638,9726,9764,1985,1943,9745,51094,1964,51092,51093
 3639,9764,9802,2027,1985,9783,51096,2006,51094,51095
 3640,9802,9840,2069,2027,9821,51098,2048,51096,51097
 3641,9840,9878,2111,2069,9859,51100,2090,51098,51099
 3642,9878,9916,2153,2111,9897,51102,2132,51100,51101
 3643,9916,9954,2195,2153,9935,51104,2174,51102,51103
 3644,9954,9992,2237,2195,9973,51106,2216,51104,51105
 3645,9992,10030,2279,2237,10011,51108,2258,51106,51107
 3646,10030,10068,2321,2279,10049,51110,2300,51108,51109
 3647,10068,10106,2363,2321,10087,51112,2342,51110,51111
 3648,10106,10144,2405,2363,10125,51114,2384,51112,51113
 3649,10144,10182,2447,2405,10163,51116,2426,51114,51115
 3650,10182,10220,2489,2447,10201,51118,2468,51116,51117
 3651,10220,10258,2531,2489,10239,51120,2510,51118,51119
 3652,10258,10296,2573,2531,10277,51122,2552,51120,51121
 3653,10296,10334,2615,2573,10315,51124,2594,51122,51123
 3654,10334,10372,2657,2615,10353,51126,2636,51124,51125
 *ELSET,ELSET=STITCHLT,GENERATE
 3592,3654,1
 **
 **STITCHMID
 **
 *ELEMENT,TYPE=S9R5
 3655,10372,10410,2699,2657,10391,51128,2678,51126,51127
 3656,10410,10448,2741,2699,10429,51130,2720,51128,51129
 3657,10448,10486,2783,2741,10467,51132,2762,51130,51131
 3658,10486,10524,2825,2783,10505,51134,2804,51132,51133
 3659,10524,10562,2867,2825,10543,51136,2846,51134,51135
 3660,10562,10600,2909,2867,10581,51138,2888,51136,51137
 3661,10600,10638,2951,2909,10619,51140,2930,51138,51139
 3662,10638,10676,2993,2951,10657,51142,2972,51140,51141
 3663,10676,10714,3035,2993,10695,51144,3014,51142,51143
 3664,10714,10752,3077,3035,10733,51146,3056,51144,51145
 3665,10752,10790,3119,3077,10771,51148,3098,51146,51147
 3666,10790,10828,3161,3119,10809,51150,3140,51148,51149
 3667,10828,10866,3203,3161,10847,51152,3182,51150,51151
 3668,10866,10904,3245,3203,10885,51154,3224,51152,51153
 3669,10904,10942,3287,3245,10923,51156,3266,51154,51155
 3670,10942,10980,3329,3287,10961,51158,3308,51156,51157
 3671,10980,11018,3371,3329,10999,51160,3350,51158,51159

3672,11018,11056,3413,3371,11037,51162,3392,51160,51161
3673,11056,11094,3455,3413,11075,51164,3434,51162,51163
3674,11094,11132,3497,3455,11113,51166,3476,51164,51165
3675,11132,11170,3539,3497,11151,51168,3518,51166,51167
3676,11170,11208,3581,3539,11189,51170,3560,51168,51169
3677,11208,11246,3623,3581,11227,51172,3602,51170,51171
3678,11246,11284,3665,3623,11265,51174,3644,51172,51173
3679,11284,11322,3707,3665,11303,51176,3686,51174,51175
3680,11322,11360,3749,3707,11341,51178,3728,51176,51177
3681,11360,11398,3791,3749,11379,51180,3770,51178,51179
3682,11398,11436,3833,3791,11417,51182,3812,51180,51181
3683,11436,11474,3875,3833,11455,51184,3854,51182,51183
3684,11474,11512,3917,3875,11493,51186,3896,51184,51185
3685,11512,11550,3959,3917,11531,51188,3938,51186,51187
3686,11550,11588,4001,3959,11569,51190,3980,51188,51189
3687,11588,11626,4043,4001,11607,51192,4022,51190,51191
3688,11626,11664,4085,4043,11645,51194,4064,51192,51193
3689,11664,11702,4127,4085,11683,51196,4106,51194,51195
3690,11702,11740,4169,4127,11721,51198,4148,51196,51197
3691,11740,11778,4211,4169,11759,51200,4190,51198,51199
3692,11778,11816,4253,4211,11797,51202,4232,51200,51201
3693,11816,11854,4295,4253,11835,51204,4274,51202,51203
3694,11854,11892,4337,4295,11873,51206,4316,51204,51205
3695,11892,11930,4379,4337,11911,51208,4358,51206,51207
3696,11930,11968,4421,4379,11949,51210,4400,51208,51209
3697,11968,12006,4463,4421,11987,51212,4442,51210,51211
3698,12006,12044,4505,4463,12025,51214,4484,51212,51213
3699,12044,12082,4547,4505,12063,51216,4526,51214,51215
3700,12082,12120,4589,4547,12101,51218,4568,51216,51217
3701,12120,12158,4631,4589,12139,51220,4610,51218,51219
3702,12158,12196,4673,4631,12177,51222,4652,51220,51221
3703,12196,12234,4715,4673,12215,51224,4694,51222,51223
3704,12234,12272,4757,4715,12253,51226,4736,51224,51225
3705,12272,12310,4799,4757,12291,51228,4778,51226,51227
3706,12310,12348,4841,4799,12329,51230,4820,51228,51229
3707,12348,12386,4883,4841,12367,51232,4862,51230,51231
3708,12386,12424,4925,4883,12405,51234,4904,51232,51233
3709,12424,12462,4967,4925,12443,51236,4946,51234,51235
3710,12462,12500,5009,4967,12481,51238,4988,51236,51237
3711,12500,12538,5051,5009,12519,51240,5030,51238,51239
3712,12538,12576,5093,5051,12557,51242,5072,51240,51241
3713,12576,12614,5135,5093,12595,51244,5114,51242,51243
3714,12614,12652,5177,5135,12633,51246,5156,51244,51245
3715,12652,12690,5219,5177,12671,51248,5198,51246,51247
3716,12690,12728,5261,5219,12709,51250,5240,51248,51249
3717,12728,12766,5303,5261,12747,51252,5282,51250,51251

*ELSET,ELSET=STITCHMID,GENERATE

3655,3717,1

**

**STITCHRT

**

*ELEMENT,TYPE=S9R5

3718,12766,12804,5345,5303,12785,51254,5324,51252,51253
3719,12804,12842,5387,5345,12823,51256,5366,51254,51255
3720,12842,12880,5429,5387,12861,51258,5408,51256,51257
3721,12880,12918,5471,5429,12899,51260,5450,51258,51259
3722,12918,12956,5513,5471,12937,51262,5492,51260,51261
3723,12956,12994,5555,5513,12975,51264,5534,51262,51263
3724,12994,13032,5597,5555,13013,51266,5576,51264,51265
3725,13032,13070,5639,5597,13051,51268,5618,51266,51267
3726,13070,13108,5681,5639,13089,51270,5660,51268,51269
3727,13108,13146,5723,5681,13127,51272,5702,51270,51271
3728,13146,13184,5765,5723,13165,51274,5744,51272,51273
3729,13184,13222,5807,5765,13203,51276,5786,51274,51275
3730,13222,13260,5849,5807,13241,51278,5828,51276,51277
3731,13260,13298,5891,5849,13279,51280,5870,51278,51279
3732,13298,13336,5933,5891,13317,51282,5912,51280,51281
3733,13336,13374,5975,5933,13355,51284,5954,51282,51283
3734,13374,13412,6017,5975,13393,51286,5996,51284,51285
3735,13412,13450,6059,6017,13431,51288,6038,51286,51287
3736,13450,13488,6101,6059,13469,51290,6080,51288,51289
3737,13488,13526,6143,6101,13507,51292,6122,51290,51291
3738,13526,13564,6185,6143,13545,51294,6164,51292,51293
3739,13564,13602,6227,6185,13583,51296,6206,51294,51295
3740,13602,13640,6269,6227,13621,51298,6248,51296,51297
3741,13640,13678,6311,6269,13659,51300,6290,51298,51299
3742,13678,13716,6353,6311,13697,51302,6332,51300,51301
3743,13716,13754,6395,6353,13735,51304,6374,51302,51303
3744,13754,13792,6437,6395,13773,51306,6416,51304,51305
3745,13792,13830,6479,6437,13811,51308,6458,51306,51307
3746,13830,13868,6521,6479,13849,51310,6500,51308,51309
3747,13868,13906,6563,6521,13887,51312,6542,51310,51311
3748,13906,13944,6605,6563,13925,51314,6584,51312,51313
3749,13944,13982,6647,6605,13963,51316,6626,51314,51315
3750,13982,14020,6689,6647,14001,51318,6668,51316,51317
3751,14020,14058,6731,6689,14039,51320,6710,51318,51319
3752,14058,14096,6773,6731,14077,51322,6752,51320,51321
3753,14096,14134,6815,6773,14115,51324,6794,51322,51323
3754,14134,14172,6857,6815,14153,51326,6836,51324,51325
3755,14172,14210,6899,6857,14191,51328,6878,51326,51327
3756,14210,14248,6941,6899,14229,51330,6920,51328,51329
3757,14248,14286,6983,6941,14267,51332,6962,51330,51331

3758,14286,14324,7025,6983,14305,51334,7004,51332,51333
 3759,14324,14362,7067,7025,14343,51336,7046,51334,51335
 3760,14362,14400,7109,7067,14381,51338,7088,51336,51337
 3761,14400,14438,7151,7109,14419,51340,7130,51338,51339
 3762,14438,14476,7193,7151,14457,51342,7172,51340,51341
 3763,14476,14514,7235,7193,14495,51344,7214,51342,51343
 3764,14514,14552,7277,7235,14533,51346,7256,51344,51345
 3765,14552,14590,7319,7277,14571,51348,7298,51346,51347
 3766,14590,14628,7361,7319,14609,51350,7340,51348,51349
 3767,14628,14666,7403,7361,14647,51352,7382,51350,51351
 3768,14666,14704,7445,7403,14685,51354,7424,51352,51353
 3769,14704,14742,7487,7445,14723,51356,7466,51354,51355
 3770,14742,14780,7529,7487,14761,51358,7508,51356,51357
 3771,14780,14818,7571,7529,14799,51360,7550,51358,51359
 3772,14818,14856,7613,7571,14837,51362,7592,51360,51361
 3773,14856,14894,7655,7613,14875,51364,7634,51362,51363
 3774,14894,14932,7697,7655,14913,51366,7676,51364,51365
 3775,14932,14970,7739,7697,14951,51368,7718,51366,51367
 3776,14970,15008,7781,7739,14989,51370,7760,51368,51369
 3777,15008,15046,7823,7781,15027,51372,7802,51370,51371
 3778,15046,15084,7865,7823,15065,51374,7844,51372,51373
 3779,15084,15122,7907,7865,15103,51376,7886,51374,51375
 3780,15122,15160,7949,7907,15141,51378,7928,51376,51377
 *ELSET,ELSET=STITCHRT,GENERATE
 3718,3780,1
 **
 **SECTION PROPERTIES
 **
 *SHELL SECTION,ELSET=FLANGELT,MATERIAL=RIGID
 1.523
 *SHELL SECTION,ELSET=WEBLT,MATERIAL=RIGID
 1.25
 *SHELL SECTION,ELSET=STITCHLT,MATERIAL=RIGID
 1.25
 *SHELL SECTION,ELSET=FLANGERT,MATERIAL=RIGID
 1.523
 *SHELL SECTION,ELSET=WEBRT,MATERIAL=RIGID
 1.25
 *SHELL SECTION,ELSET=STITCHRT,MATERIAL=RIGID
 1.25
 *SHELL SECTION,ELSET=FLANGEMID,MATERIAL=STEEL
 1.523
 *SHELL SECTION,ELSET=WEBMID,MATERIAL=STEEL
 1.25
 *SHELL SECTION,ELSET=STITCHMID,MATERIAL=STEEL
 1.25

```

**
**MATERIAL PROPERTIES
**
*MATERIAL,NAME=RIGID
*ELASTIC
2.90E+05,0.3
*MATERIAL,NAME=STEEL
*ELASTIC
2.90E+04,0.3
*PLASTIC
50,0
51.345,0.00922948
75,0.0557238
80,0.090034
**
**BOUNDARY CONDITIONS
**
*BOUNDARY
FLPINLT,1
FLPINLT,2
FLPINLT,3
FLPINRT,2
FLPINRT,3
WEBPINLT,2
WEBPINRT,2
MIDPINLT,2
MIDPINRT,2
**
**ANALYSIS,INPUT FILE
**
*STEP
*BUCKLE
10,,20,80
*CLOAD
LTLOAD,3,1
RTLOAD,3,1
*RESTART,WRITE,FREQUENCY=1
*NODE FILE,LAST MODE=1,GLOBAL=YES
U
*ELPRINT,FREQUENCY=0
*END STEP

```

***HEADING – MATERIAL PROPERTIES FILE**

ALTER tw WHILE bf/d=2.0

RIGID ENDS

S9R5 ELEMENTS

APPROXIMATELY 1" ELEMENTS

ACTUAL WTEE DIMENSIONS

**

****INITIAL CONDITIONS**

**

***NODE**

1,0,0,0

21,0,28,0

2647,176.573,0,0

2667,176.573,28,0

5293,353.146,0,0

5313,353.146,28,0

7939,529.719,0,0

7959,529.719,28,0

7960,0,14,-14.0005

7978,0,14,-1.288

10354,176.573,14,-14.0005

10372,176.573,14,-1.288

12748,353.146,14,-14.0005

12766,353.146,14,-1.288

15142,529.719,14,-14.0005

15160,529.719,14,-1.288

51000,0,14,-0.644

51378,529.719,14,-0.644

**

****NODE DEFINITION**

**

***NGEN,NSET=BFLANGELT**

1,2647,21

***NGEN,NSET=TFLANGELT**

21,2667,21

***NGEN,NSET=BFLANGEMID**

2647,5293,21

***NGEN,NSET=TFLANGEMID**

2667,5313,21

***NGEN,NSET=BFLANGERT**

5293,7939,21

***NGEN,NSET=TFLANGERT**

5313,7959,21

***NGEN,NSET=BWEBLT**

7960,10354,19

```

*NGEN,NSET=TWEBLT
7978,10372,19
*NGEN,NSET=BWEBMID
10354,12748,19
*NGEN,NSET=TWEBMID
10372,12766,19
*NGEN,NSET=BWEBRT
12748,15142,19
*NGEN,NSET=TWEBRT
12766,15160,19
*NGEN,NSET=STITCH
51000,51378,1
**
**FILL NODES
**
*NFill,NSET=FLANGELT
BFLANGELT,TFLANGELT,20
*NFill,NSET=FLANGEMID
BFLANGEMID,TFLANGEMID,20
*NFill,NSET=FLANGERT
BFLANGERT,TFLANGERT,20
*NFill,NSET=WEBLT
BWEBLT,TWEBLT,18
*NFill,NSET=WEBMID
BWEBMID,TWEBMID,18
*NFill,NSET=WEBRT
BWEBRT,TWEBRT,18
**
**CONSTRAINT SETS
**
*NSET,NSET=FLPINLT,GENERATE
1,21,1
*NSET,NSET=FLPINRT,GENERATE
7939,7959,1
*NSET,NSET=WEBPINLT,GENERATE
7960,7978,1
*NSET,NSET=WEBPINRT,GENERATE
15142,15160,1
*NSET,NSET=MIDPINLT,GENERATE
10354,10372,1
*NSET,NSET=MIDPINRT,GENERATE
12748,12766,1
*NSET,NSET=LTLOAD,GENERATE
2647,2667,1
*NSET,NSET=RTLOAD,GENERATE
5293,5313,1

```

```

**
**FLANGELT
**
*ELEMENT,TYPE=S9R5
1,1,43,45,3,22,44,24,2,23
*ELGEN,ELSET=FLANGELT
1,63,42,1,10,2,189
**
**FLANGEMID
**
*ELEMENT,TYPE=S9R5
64,2647,2689,2691,2649,2668,2690,2670,2648,2669
*ELGEN,ELSET=FLANGEMID
64,63,42,1,10,2,189
**
**FLANGERT
**
*ELEMENT,TYPE=S9R5
127,5293,5335,5337,5295,5314,5336,5316,5294,5315
*ELGEN,ELSET=FLANGERT
127,63,42,1,10,2,189
**
**WEBLT
**
*ELEMENT,TYPE=S9R5
1891,7960,7998,8000,7962,7979,7999,7981,7961,7980
*ELGEN,ELSET=WEBLT
1891,63,38,1,9,2,189
**
**WEBMID
**
*ELEMENT,TYPE=S9R5
1954,10354,10392,10394,10356,10373,10393,10375,10355,10374
*ELGEN,ELSET=WEBMID
1954,63,38,1,9,2,189
**
**WEBRT
**
*ELEMENT,TYPE=S9R5
2017,12748,12786,12788,12750,12767,12787,12769,12749,12768
*ELGEN,ELSET=WEBRT
2017,63,38,1,9,2,189

```

**

**STITCHLT

**

*ELEMENT,TYPE=S9R5

3592,7978,8016,53,11,7997,51002,32,51000,51001
3593,8016,8054,95,53,8035,51004,74,51002,51003
3594,8054,8092,137,95,8073,51006,116,51004,51005
3595,8092,8130,179,137,8111,51008,158,51006,51007
3596,8130,8168,221,179,8149,51010,200,51008,51009
3597,8168,8206,263,221,8187,51012,242,51010,51011
3598,8206,8244,305,263,8225,51014,284,51012,51013
3599,8244,8282,347,305,8263,51016,326,51014,51015
3600,8282,8320,389,347,8301,51018,368,51016,51017
3601,8320,8358,431,389,8339,51020,410,51018,51019
3602,8358,8396,473,431,8377,51022,452,51020,51021
3603,8396,8434,515,473,8415,51024,494,51022,51023
3604,8434,8472,557,515,8453,51026,536,51024,51025
3605,8472,8510,599,557,8491,51028,578,51026,51027
3606,8510,8548,641,599,8529,51030,620,51028,51029
3607,8548,8586,683,641,8567,51032,662,51030,51031
3608,8586,8624,725,683,8605,51034,704,51032,51033
3609,8624,8662,767,725,8643,51036,746,51034,51035
3610,8662,8700,809,767,8681,51038,788,51036,51037
3611,8700,8738,851,809,8719,51040,830,51038,51039
3612,8738,8776,893,851,8757,51042,872,51040,51041
3613,8776,8814,935,893,8795,51044,914,51042,51043
3614,8814,8852,977,935,8833,51046,956,51044,51045
3615,8852,8890,1019,977,8871,51048,998,51046,51047
3616,8890,8928,1061,1019,8909,51050,1040,51048,51049
3617,8928,8966,1103,1061,8947,51052,1082,51050,51051
3618,8966,9004,1145,1103,8985,51054,1124,51052,51053
3619,9004,9042,1187,1145,9023,51056,1166,51054,51055
3620,9042,9080,1229,1187,9061,51058,1208,51056,51057
3621,9080,9118,1271,1229,9099,51060,1250,51058,51059
3622,9118,9156,1313,1271,9137,51062,1292,51060,51061
3623,9156,9194,1355,1313,9175,51064,1334,51062,51063
3624,9194,9232,1397,1355,9213,51066,1376,51064,51065
3625,9232,9270,1439,1397,9251,51068,1418,51066,51067
3626,9270,9308,1481,1439,9289,51070,1460,51068,51069
3627,9308,9346,1523,1481,9327,51072,1502,51070,51071
3628,9346,9384,1565,1523,9365,51074,1544,51072,51073
3629,9384,9422,1607,1565,9403,51076,1586,51074,51075
3630,9422,9460,1649,1607,9441,51078,1628,51076,51077
3631,9460,9498,1691,1649,9479,51080,1670,51078,51079
3632,9498,9536,1733,1691,9517,51082,1712,51080,51081
3633,9536,9574,1775,1733,9555,51084,1754,51082,51083

3634,9574,9612,1817,1775,9593,51086,1796,51084,51085
 3635,9612,9650,1859,1817,9631,51088,1838,51086,51087
 3636,9650,9688,1901,1859,9669,51090,1880,51088,51089
 3637,9688,9726,1943,1901,9707,51092,1922,51090,51091
 3638,9726,9764,1985,1943,9745,51094,1964,51092,51093
 3639,9764,9802,2027,1985,9783,51096,2006,51094,51095
 3640,9802,9840,2069,2027,9821,51098,2048,51096,51097
 3641,9840,9878,2111,2069,9859,51100,2090,51098,51099
 3642,9878,9916,2153,2111,9897,51102,2132,51100,51101
 3643,9916,9954,2195,2153,9935,51104,2174,51102,51103
 3644,9954,9992,2237,2195,9973,51106,2216,51104,51105
 3645,9992,10030,2279,2237,10011,51108,2258,51106,51107
 3646,10030,10068,2321,2279,10049,51110,2300,51108,51109
 3647,10068,10106,2363,2321,10087,51112,2342,51110,51111
 3648,10106,10144,2405,2363,10125,51114,2384,51112,51113
 3649,10144,10182,2447,2405,10163,51116,2426,51114,51115
 3650,10182,10220,2489,2447,10201,51118,2468,51116,51117
 3651,10220,10258,2531,2489,10239,51120,2510,51118,51119
 3652,10258,10296,2573,2531,10277,51122,2552,51120,51121
 3653,10296,10334,2615,2573,10315,51124,2594,51122,51123
 3654,10334,10372,2657,2615,10353,51126,2636,51124,51125
 *ELSET,ELSET=STITCHLT,GENERATE
 3592,3654,1
 **
 **STITCHMID
 **
 *ELEMENT,TYPE=S9R5
 3655,10372,10410,2699,2657,10391,51128,2678,51126,51127
 3656,10410,10448,2741,2699,10429,51130,2720,51128,51129
 3657,10448,10486,2783,2741,10467,51132,2762,51130,51131
 3658,10486,10524,2825,2783,10505,51134,2804,51132,51133
 3659,10524,10562,2867,2825,10543,51136,2846,51134,51135
 3660,10562,10600,2909,2867,10581,51138,2888,51136,51137
 3661,10600,10638,2951,2909,10619,51140,2930,51138,51139
 3662,10638,10676,2993,2951,10657,51142,2972,51140,51141
 3663,10676,10714,3035,2993,10695,51144,3014,51142,51143
 3664,10714,10752,3077,3035,10733,51146,3056,51144,51145
 3665,10752,10790,3119,3077,10771,51148,3098,51146,51147
 3666,10790,10828,3161,3119,10809,51150,3140,51148,51149
 3667,10828,10866,3203,3161,10847,51152,3182,51150,51151
 3668,10866,10904,3245,3203,10885,51154,3224,51152,51153
 3669,10904,10942,3287,3245,10923,51156,3266,51154,51155
 3670,10942,10980,3329,3287,10961,51158,3308,51156,51157
 3671,10980,11018,3371,3329,10999,51160,3350,51158,51159
 3672,11018,11056,3413,3371,11037,51162,3392,51160,51161
 3673,11056,11094,3455,3413,11075,51164,3434,51162,51163

3674,11094,11132,3497,3455,11113,51166,3476,51164,51165
3675,11132,11170,3539,3497,11151,51168,3518,51166,51167
3676,11170,11208,3581,3539,11189,51170,3560,51168,51169
3677,11208,11246,3623,3581,11227,51172,3602,51170,51171
3678,11246,11284,3665,3623,11265,51174,3644,51172,51173
3679,11284,11322,3707,3665,11303,51176,3686,51174,51175
3680,11322,11360,3749,3707,11341,51178,3728,51176,51177
3681,11360,11398,3791,3749,11379,51180,3770,51178,51179
3682,11398,11436,3833,3791,11417,51182,3812,51180,51181
3683,11436,11474,3875,3833,11455,51184,3854,51182,51183
3684,11474,11512,3917,3875,11493,51186,3896,51184,51185
3685,11512,11550,3959,3917,11531,51188,3938,51186,51187
3686,11550,11588,4001,3959,11569,51190,3980,51188,51189
3687,11588,11626,4043,4001,11607,51192,4022,51190,51191
3688,11626,11664,4085,4043,11645,51194,4064,51192,51193
3689,11664,11702,4127,4085,11683,51196,4106,51194,51195
3690,11702,11740,4169,4127,11721,51198,4148,51196,51197
3691,11740,11778,4211,4169,11759,51200,4190,51198,51199
3692,11778,11816,4253,4211,11797,51202,4232,51200,51201
3693,11816,11854,4295,4253,11835,51204,4274,51202,51203
3694,11854,11892,4337,4295,11873,51206,4316,51204,51205
3695,11892,11930,4379,4337,11911,51208,4358,51206,51207
3696,11930,11968,4421,4379,11949,51210,4400,51208,51209
3697,11968,12006,4463,4421,11987,51212,4442,51210,51211
3698,12006,12044,4505,4463,12025,51214,4484,51212,51213
3699,12044,12082,4547,4505,12063,51216,4526,51214,51215
3700,12082,12120,4589,4547,12101,51218,4568,51216,51217
3701,12120,12158,4631,4589,12139,51220,4610,51218,51219
3702,12158,12196,4673,4631,12177,51222,4652,51220,51221
3703,12196,12234,4715,4673,12215,51224,4694,51222,51223
3704,12234,12272,4757,4715,12253,51226,4736,51224,51225
3705,12272,12310,4799,4757,12291,51228,4778,51226,51227
3706,12310,12348,4841,4799,12329,51230,4820,51228,51229
3707,12348,12386,4883,4841,12367,51232,4862,51230,51231
3708,12386,12424,4925,4883,12405,51234,4904,51232,51233
3709,12424,12462,4967,4925,12443,51236,4946,51234,51235
3710,12462,12500,5009,4967,12481,51238,4988,51236,51237
3711,12500,12538,5051,5009,12519,51240,5030,51238,51239
3712,12538,12576,5093,5051,12557,51242,5072,51240,51241
3713,12576,12614,5135,5093,12595,51244,5114,51242,51243
3714,12614,12652,5177,5135,12633,51246,5156,51244,51245
3715,12652,12690,5219,5177,12671,51248,5198,51246,51247
3716,12690,12728,5261,5219,12709,51250,5240,51248,51249
3717,12728,12766,5303,5261,12747,51252,5282,51250,51251
*ELSET,ELSET=STITCHMID,GENERATE
3655,3717,1

**

**STITCHRT

**

*ELEMENT,TYPE=S9R5

3718,12766,12804,5345,5303,12785,51254,5324,51252,51253
3719,12804,12842,5387,5345,12823,51256,5366,51254,51255
3720,12842,12880,5429,5387,12861,51258,5408,51256,51257
3721,12880,12918,5471,5429,12899,51260,5450,51258,51259
3722,12918,12956,5513,5471,12937,51262,5492,51260,51261
3723,12956,12994,5555,5513,12975,51264,5534,51262,51263
3724,12994,13032,5597,5555,13013,51266,5576,51264,51265
3725,13032,13070,5639,5597,13051,51268,5618,51266,51267
3726,13070,13108,5681,5639,13089,51270,5660,51268,51269
3727,13108,13146,5723,5681,13127,51272,5702,51270,51271
3728,13146,13184,5765,5723,13165,51274,5744,51272,51273
3729,13184,13222,5807,5765,13203,51276,5786,51274,51275
3730,13222,13260,5849,5807,13241,51278,5828,51276,51277
3731,13260,13298,5891,5849,13279,51280,5870,51278,51279
3732,13298,13336,5933,5891,13317,51282,5912,51280,51281
3733,13336,13374,5975,5933,13355,51284,5954,51282,51283
3734,13374,13412,6017,5975,13393,51286,5996,51284,51285
3735,13412,13450,6059,6017,13431,51288,6038,51286,51287
3736,13450,13488,6101,6059,13469,51290,6080,51288,51289
3737,13488,13526,6143,6101,13507,51292,6122,51290,51291
3738,13526,13564,6185,6143,13545,51294,6164,51292,51293
3739,13564,13602,6227,6185,13583,51296,6206,51294,51295
3740,13602,13640,6269,6227,13621,51298,6248,51296,51297
3741,13640,13678,6311,6269,13659,51300,6290,51298,51299
3742,13678,13716,6353,6311,13697,51302,6332,51300,51301
3743,13716,13754,6395,6353,13735,51304,6374,51302,51303
3744,13754,13792,6437,6395,13773,51306,6416,51304,51305
3745,13792,13830,6479,6437,13811,51308,6458,51306,51307
3746,13830,13868,6521,6479,13849,51310,6500,51308,51309
3747,13868,13906,6563,6521,13887,51312,6542,51310,51311
3748,13906,13944,6605,6563,13925,51314,6584,51312,51313
3749,13944,13982,6647,6605,13963,51316,6626,51314,51315
3750,13982,14020,6689,6647,14001,51318,6668,51316,51317
3751,14020,14058,6731,6689,14039,51320,6710,51318,51319
3752,14058,14096,6773,6731,14077,51322,6752,51320,51321
3753,14096,14134,6815,6773,14115,51324,6794,51322,51323
3754,14134,14172,6857,6815,14153,51326,6836,51324,51325
3755,14172,14210,6899,6857,14191,51328,6878,51326,51327
3756,14210,14248,6941,6899,14229,51330,6920,51328,51329
3757,14248,14286,6983,6941,14267,51332,6962,51330,51331
3758,14286,14324,7025,6983,14305,51334,7004,51332,51333
3759,14324,14362,7067,7025,14343,51336,7046,51334,51335

3760,14362,14400,7109,7067,14381,51338,7088,51336,51337
 3761,14400,14438,7151,7109,14419,51340,7130,51338,51339
 3762,14438,14476,7193,7151,14457,51342,7172,51340,51341
 3763,14476,14514,7235,7193,14495,51344,7214,51342,51343
 3764,14514,14552,7277,7235,14533,51346,7256,51344,51345
 3765,14552,14590,7319,7277,14571,51348,7298,51346,51347
 3766,14590,14628,7361,7319,14609,51350,7340,51348,51349
 3767,14628,14666,7403,7361,14647,51352,7382,51350,51351
 3768,14666,14704,7445,7403,14685,51354,7424,51352,51353
 3769,14704,14742,7487,7445,14723,51356,7466,51354,51355
 3770,14742,14780,7529,7487,14761,51358,7508,51356,51357
 3771,14780,14818,7571,7529,14799,51360,7550,51358,51359
 3772,14818,14856,7613,7571,14837,51362,7592,51360,51361
 3773,14856,14894,7655,7613,14875,51364,7634,51362,51363
 3774,14894,14932,7697,7655,14913,51366,7676,51364,51365
 3775,14932,14970,7739,7697,14951,51368,7718,51366,51367
 3776,14970,15008,7781,7739,14989,51370,7760,51368,51369
 3777,15008,15046,7823,7781,15027,51372,7802,51370,51371
 3778,15046,15084,7865,7823,15065,51374,7844,51372,51373
 3779,15084,15122,7907,7865,15103,51376,7886,51374,51375
 3780,15122,15160,7949,7907,15141,51378,7928,51376,51377
 *ELSET,ELSET=STITCHRT,GENERATE
 3718,3780,1
 **
 **SECTION PROPERTIES
 **
 *SHELL SECTION,ELSET=FLANGELT,MATERIAL=RIGID
 1.523
 *SHELL SECTION,ELSET=WEBLT,MATERIAL=RIGID
 1.25
 *SHELL SECTION,ELSET=STITCHLT,MATERIAL=RIGID
 1.25
 *SHELL SECTION,ELSET=FLANGERT,MATERIAL=RIGID
 1.523
 *SHELL SECTION,ELSET=WEBRT,MATERIAL=RIGID
 1.25
 *SHELL SECTION,ELSET=STITCHRT,MATERIAL=RIGID
 1.25
 *SHELL SECTION,ELSET=FLANGEMID,MATERIAL=STEEL
 1.523
 *SHELL SECTION,ELSET=WEBMID,MATERIAL=STEEL
 1.25
 *SHELL SECTION,ELSET=STITCHMID,MATERIAL=STEEL
 1.25

```

**
**MATERIAL PROPERTIES
**
*MATERIAL,NAME=RIGID
*ELASTIC
2.90E+05,0.3
*MATERIAL,NAME=STEEL
*ELASTIC
2.90E+04,0.3
*PLASTIC
50,0
51.345,0.00922948
75,0.0557238
80,0.090034
**
**BOUNDARY CONDITIONS
**
*BOUNDARY
FLPINLT,1
FLPINLT,2
FLPINLT,3
FLPINRT,2
FLPINRT,3
WEBPINLT,2
WEBPINRT,2
MIDPINLT,2
MIDPINRT,2
**
**ANALYSIS,MATERIAL PROPERTIES
**
*IMPERFECTION,FILE=var2_1.25_imp,STEP=1
1,0.176573
*STEP,NLGEOM,INC=50
*STATIC,RIKS
0.001,1
*CLOAD
LTLOAD,3,1
RTLOAD,3,1
*RESTART,WRITE,FREQUENCY=1
*END STEP

```

File used to obtain the reactions at the end of the beam (RF3), rotation at the centerline of the end of the beam (U3), and the overall deflection at the center point of the beam at the bottom of the stem (UR2).

```
read curve, name=load1, variable=RF3, node=1
read curve, name=load2, variable=RF3, node=2
read curve, name=load3, variable=RF3, node=3
read curve, name=load4, variable=RF3, node=4
read curve, name=load5, variable=RF3, node=5
read curve, name=load6, variable=RF3, node=6
read curve, name=load7, variable=RF3, node=7
read curve, name=load8, variable=RF3, node=8
read curve, name=load9, variable=RF3, node=9
read curve, name=load10, variable=RF3, node=10
read curve, name=load11, variable=RF3, node=11
read curve, name=load12, variable=RF3, node=12
read curve, name=load13, variable=RF3, node=13
read curve, name=load14, variable=RF3, node=14
read curve, name=load15, variable=RF3, node=15
read curve, name=load16, variable=RF3, node=16
read curve, name=load17, variable=RF3, node=17
read curve, name=load18, variable=RF3, node=18
read curve, name=load19, variable=RF3, node=19
read curve, name=load20, variable=RF3, node=20
read curve, name=load21, variable=RF3, node=21
read curve, name=defl, variable=U3, node=3981
read curve, name=rot, variable=UR2, node=11
print curve, all curves
```

APPENDIX B

Output From ABAQUS

Upon completion of an analysis of a WTEE beam, employing ABAQUS, the output is analyzed. From this output, the moment versus rotation curves may be plotted, and thus, the rotation capacity calculated. A graph of the moment vs. rotation for each of the cases of b_f/d is shown appendix Section B1.

Illustrative figures from ABAQUS are presented in Section B2. These figures include a typical seed imperfection that is applied to each of the WTEE beams in this research, a typical deflected shape after the WTEE beam has been loaded, and various beams with the von Mises stresses shown through the use of shading.

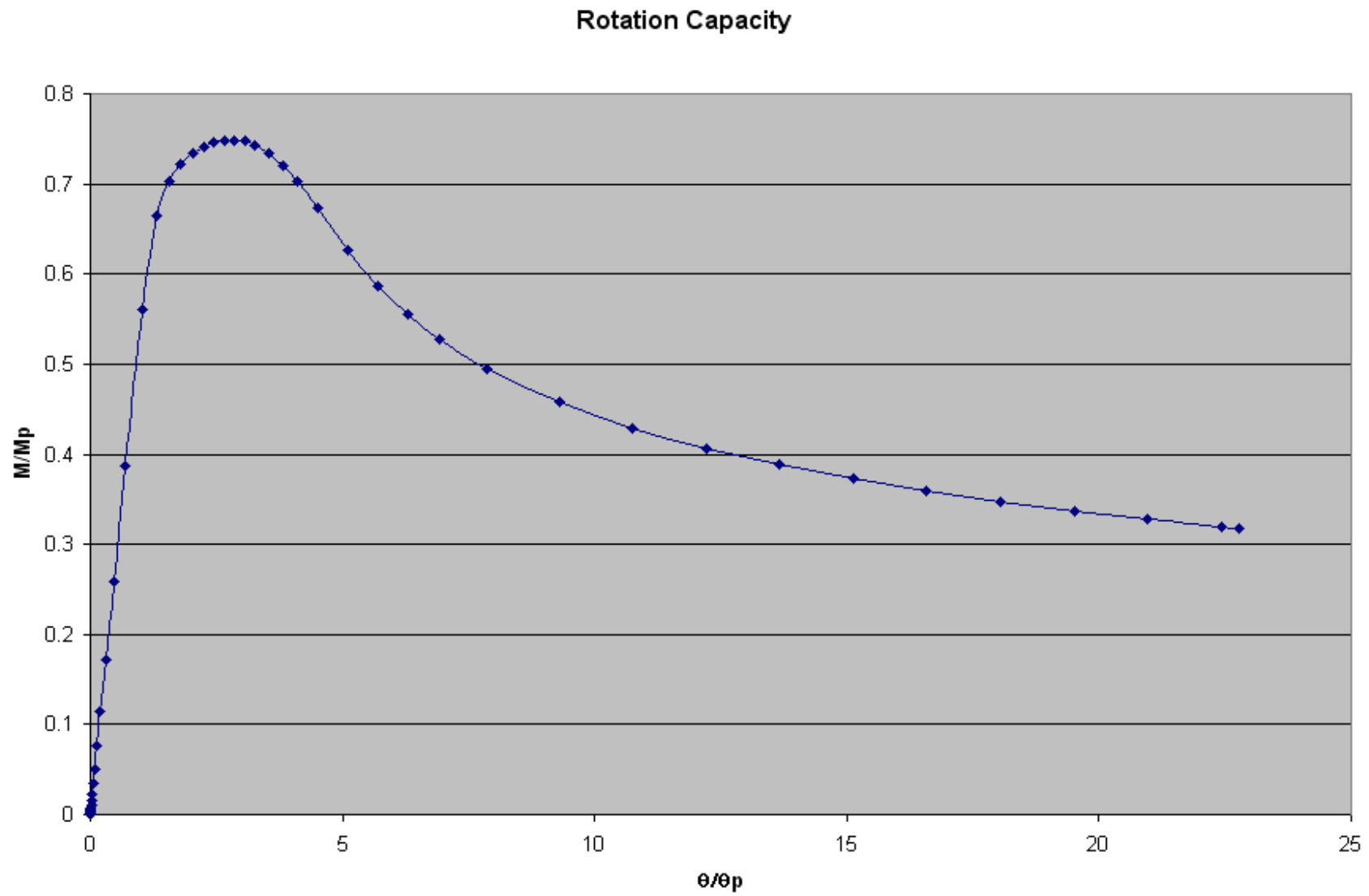


Figure B1 Moment vs. Rotation for WTEE $b_f/d=0.6$, $t_w=1.0''$

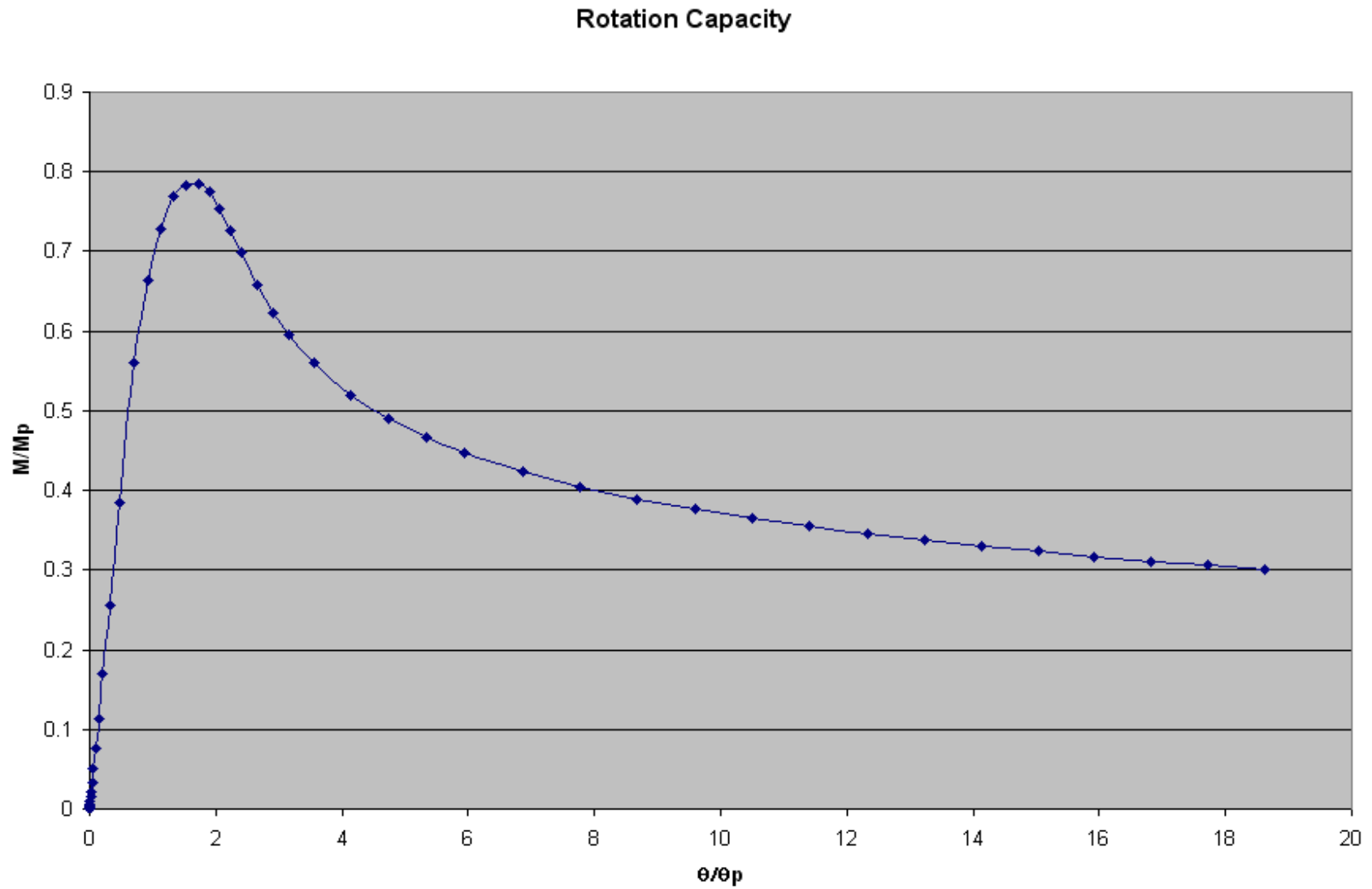


Figure B2 Moment vs. Rotation for WTEE $b_f/d=0.8$, $t_w=1.0''$

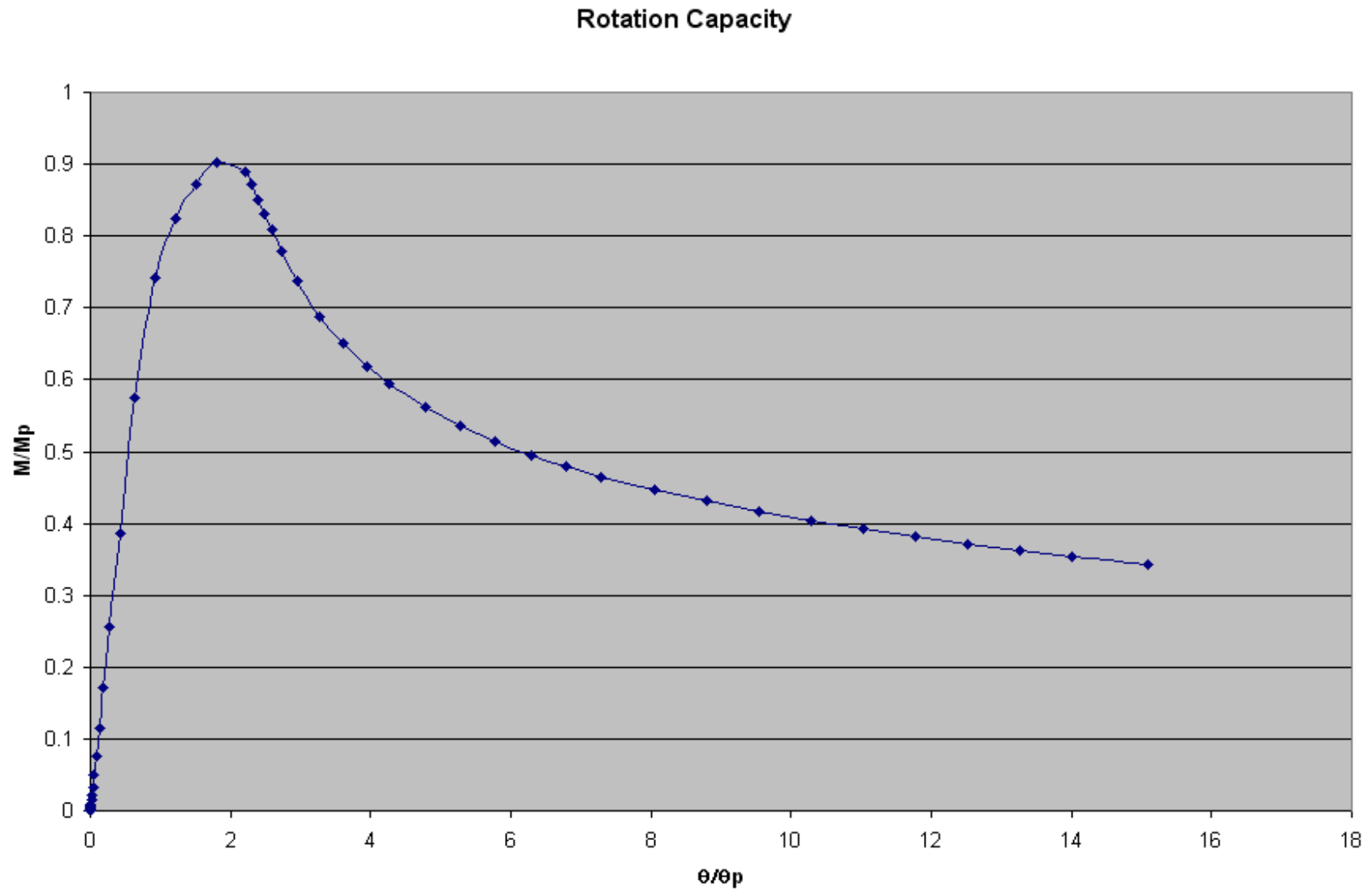


Figure B3 Moment vs. Rotation for WTEE $b_f/d=1.0$, $t_w=1.0''$

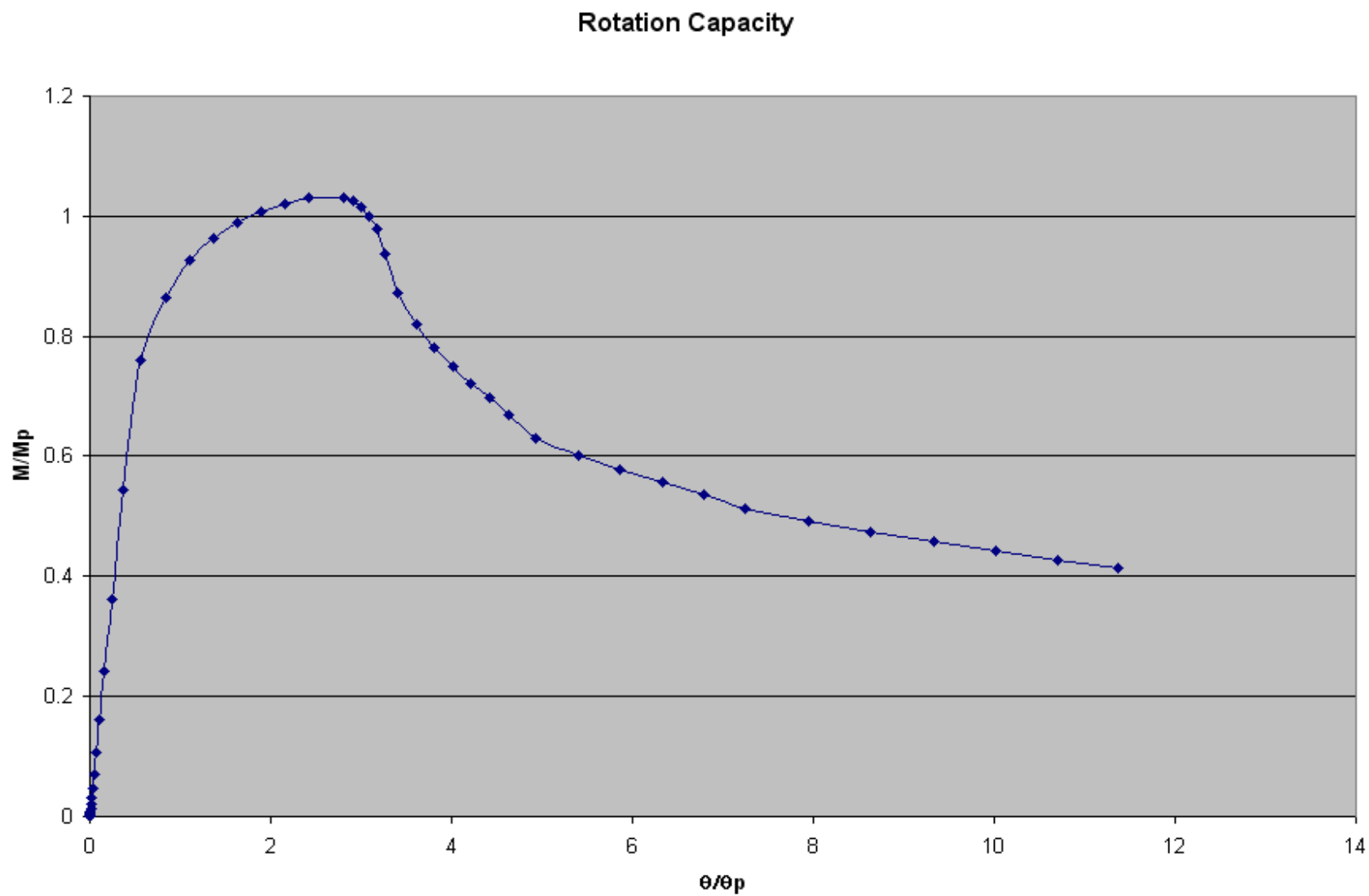


Figure B4 Moment vs. Rotation for WTEE $b_f/d=1.2$, $t_w=1.0''$

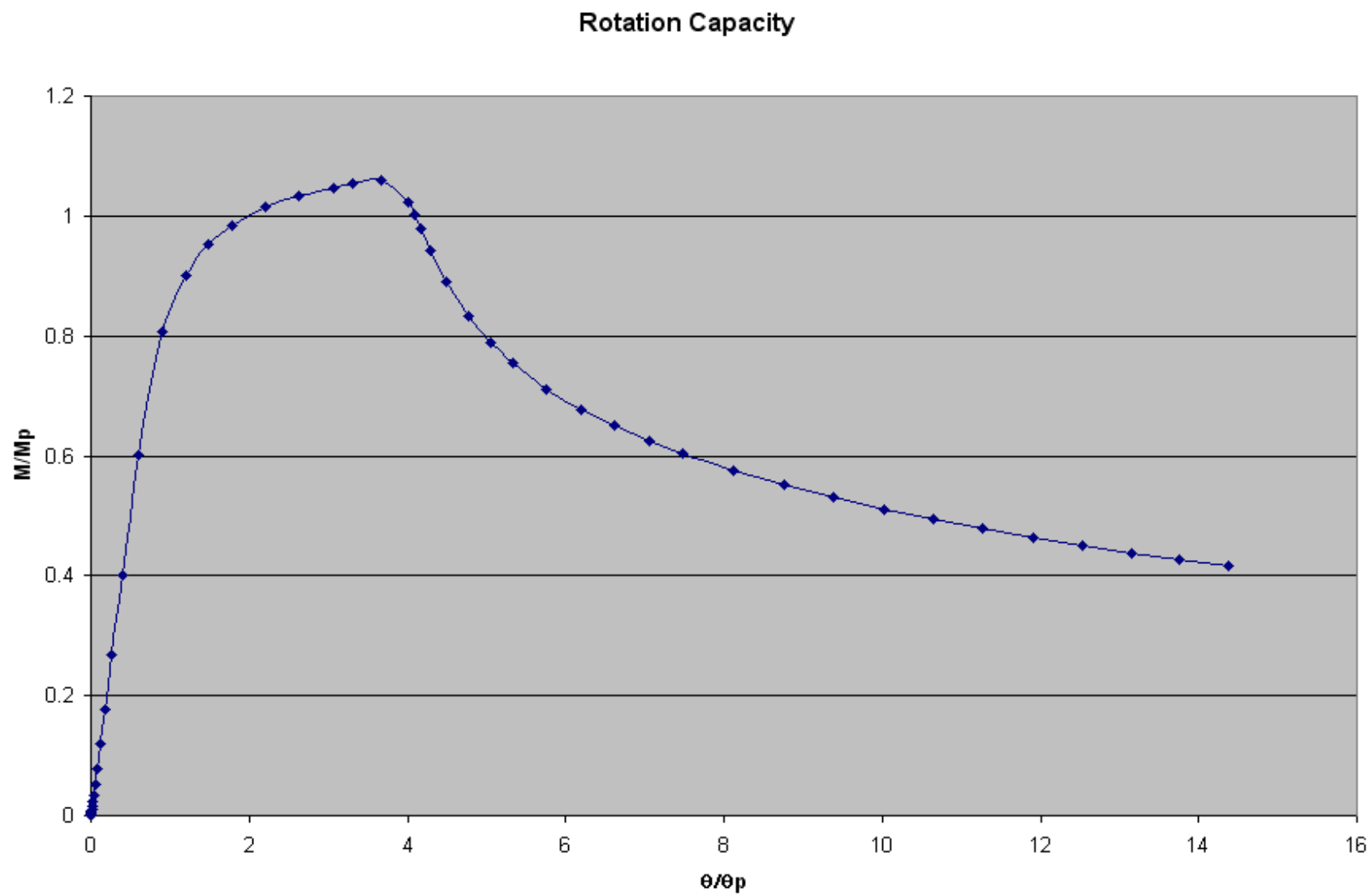


Figure B5 Moment vs. Rotation for WTEE $b_f/d=1.4$, $t_w=1.1''$

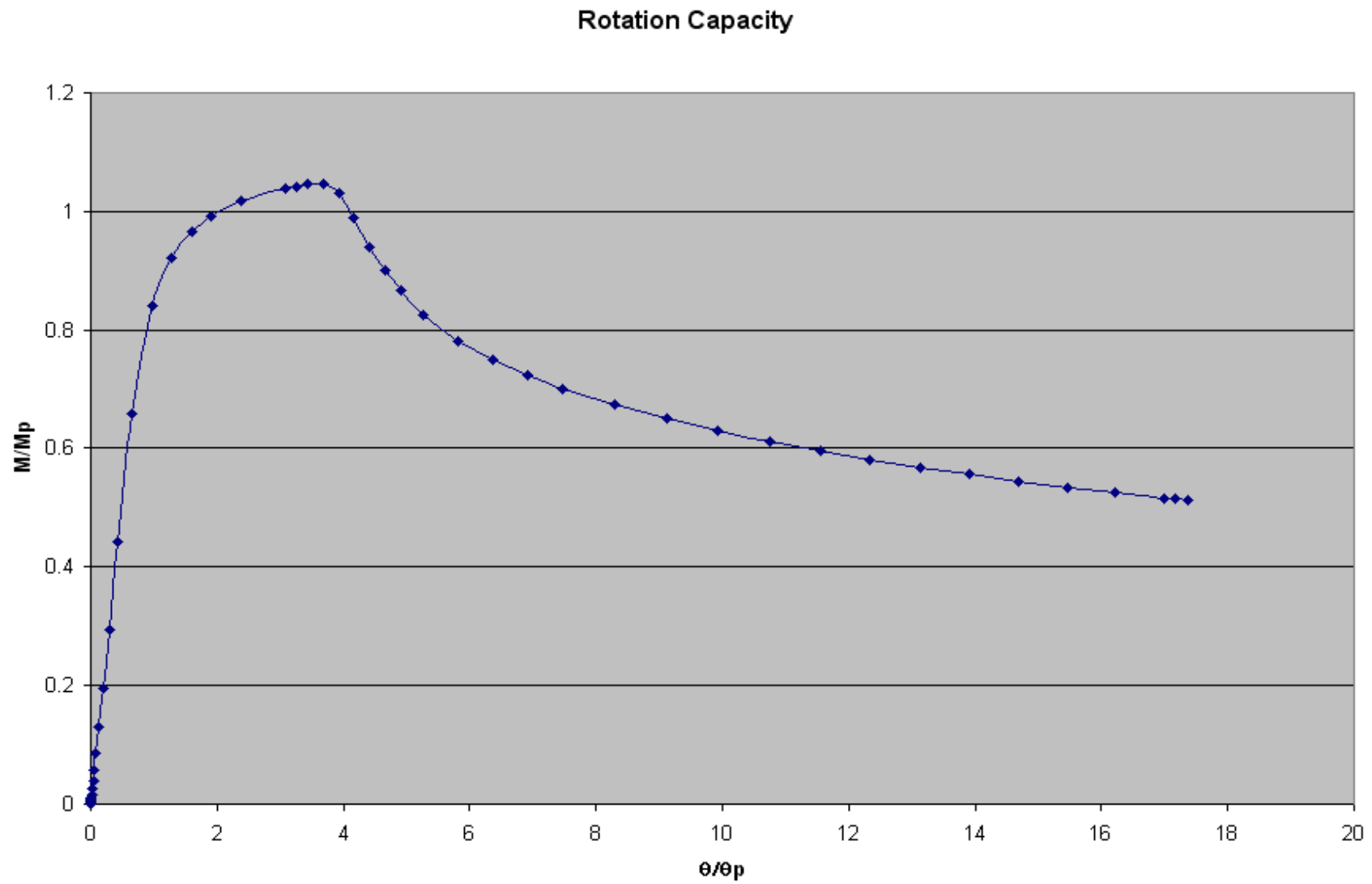


Figure B6 Moment vs. Rotation for WTEE $b_f/d=1.6$, $t_w=1.17''$

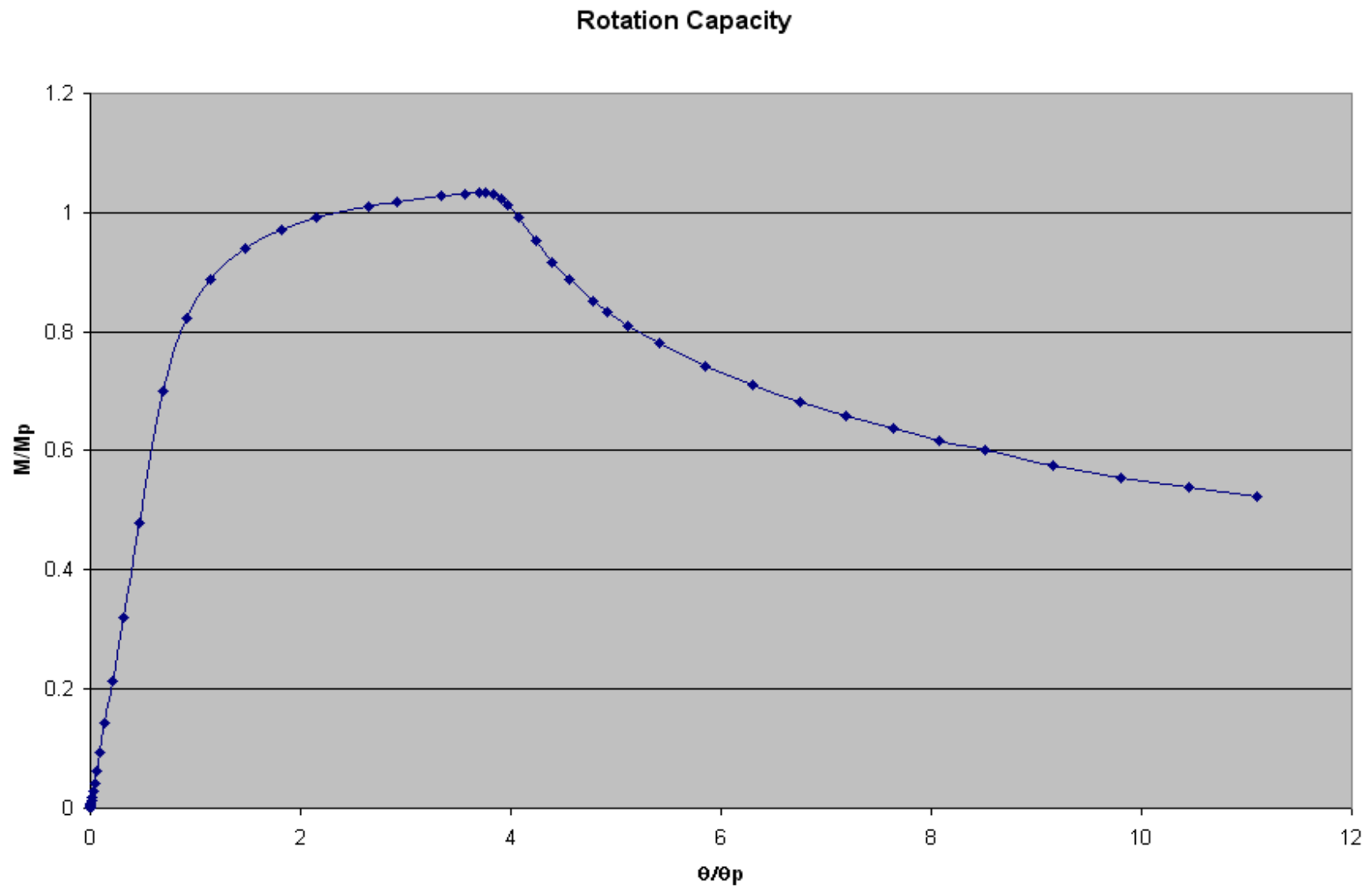


Figure B7 Moment vs. Rotation for WTEE $b_f/d=1.8$, $t_w=1.22''$

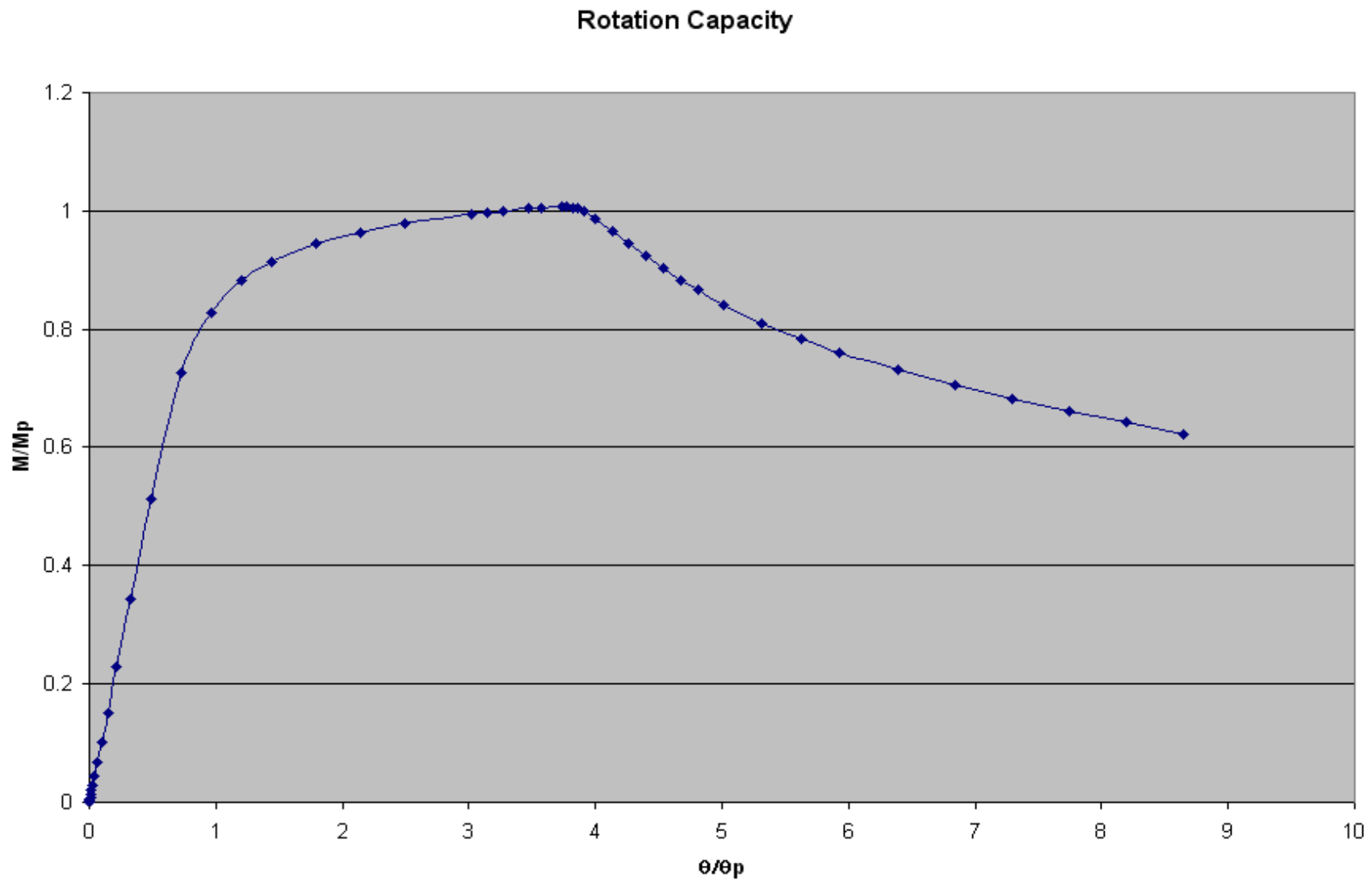


Figure B8 Moment vs. Rotation for WTEE $b_f/d=2.0$, $t_w=1.25''$

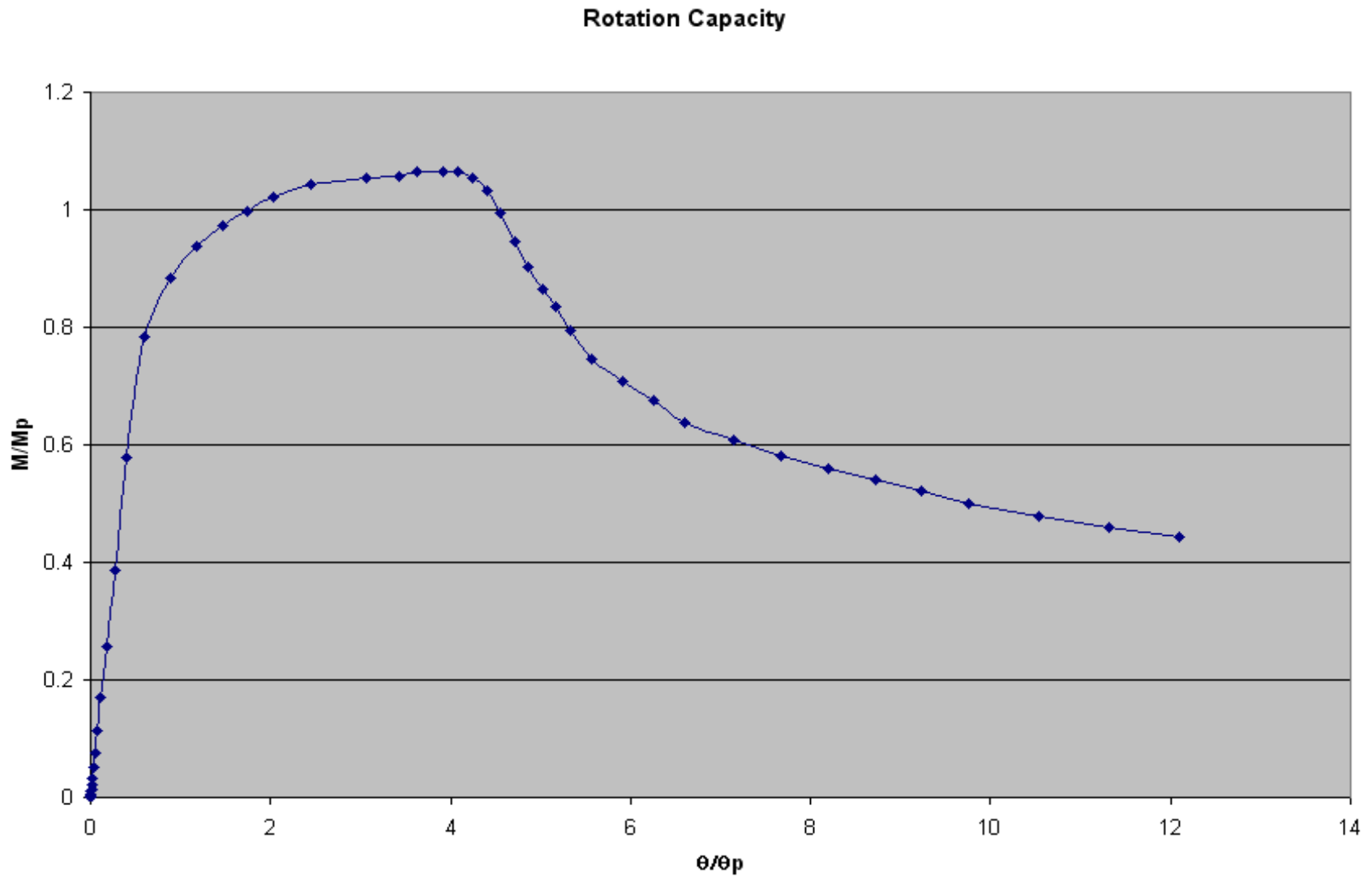


Figure B9 Moment vs. Rotation for WTEE $b_f/d=1.4$, $t_w=1.1''$, $\lambda_p = 1.2(65/\sqrt{F_y})$

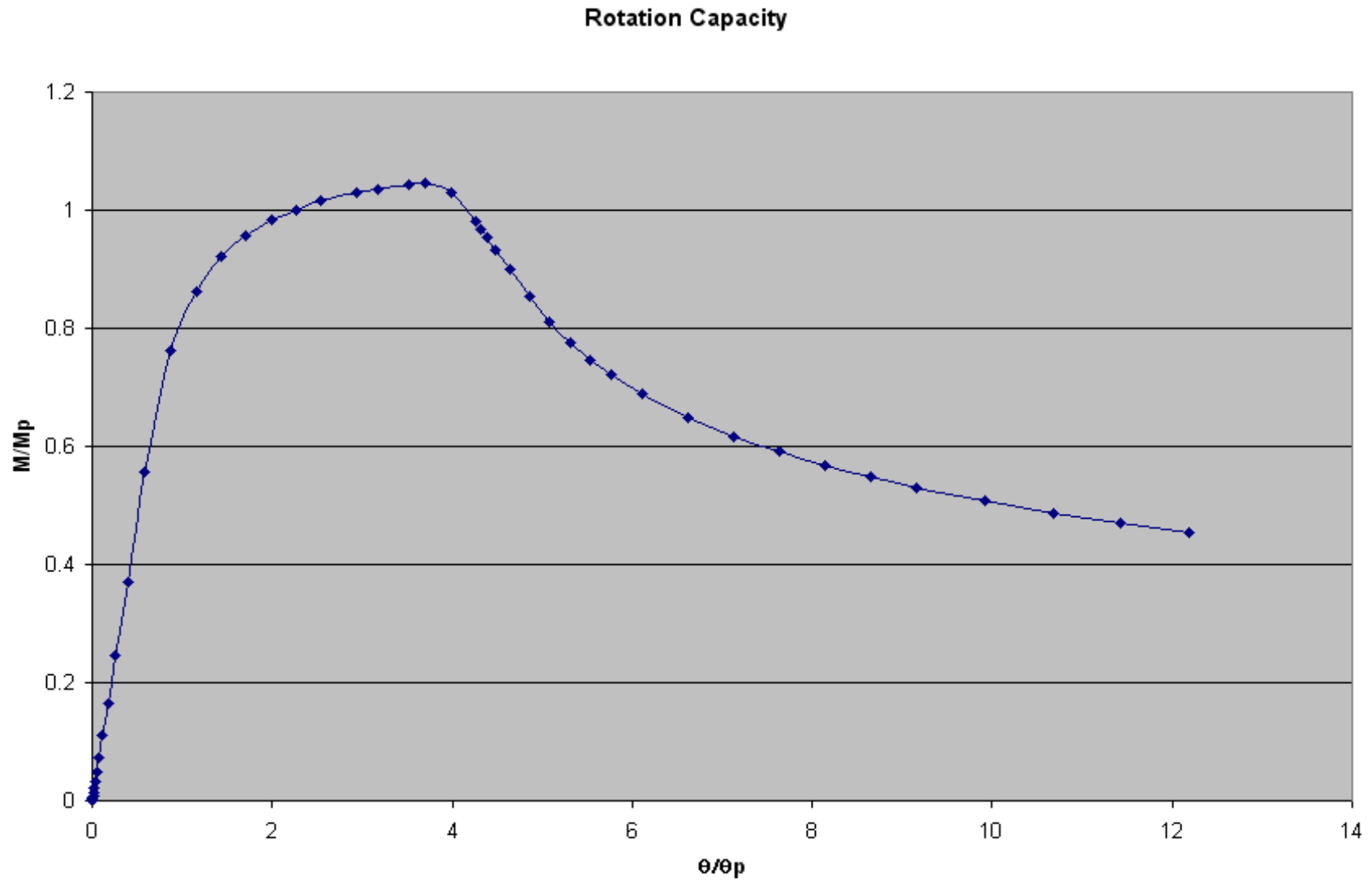


Figure B10 Moment vs. Rotation for WTEE $b_f/d=1.4$, $t_w=1.1''$, $\lambda_p = 1.4(65/\sqrt{F_y})$

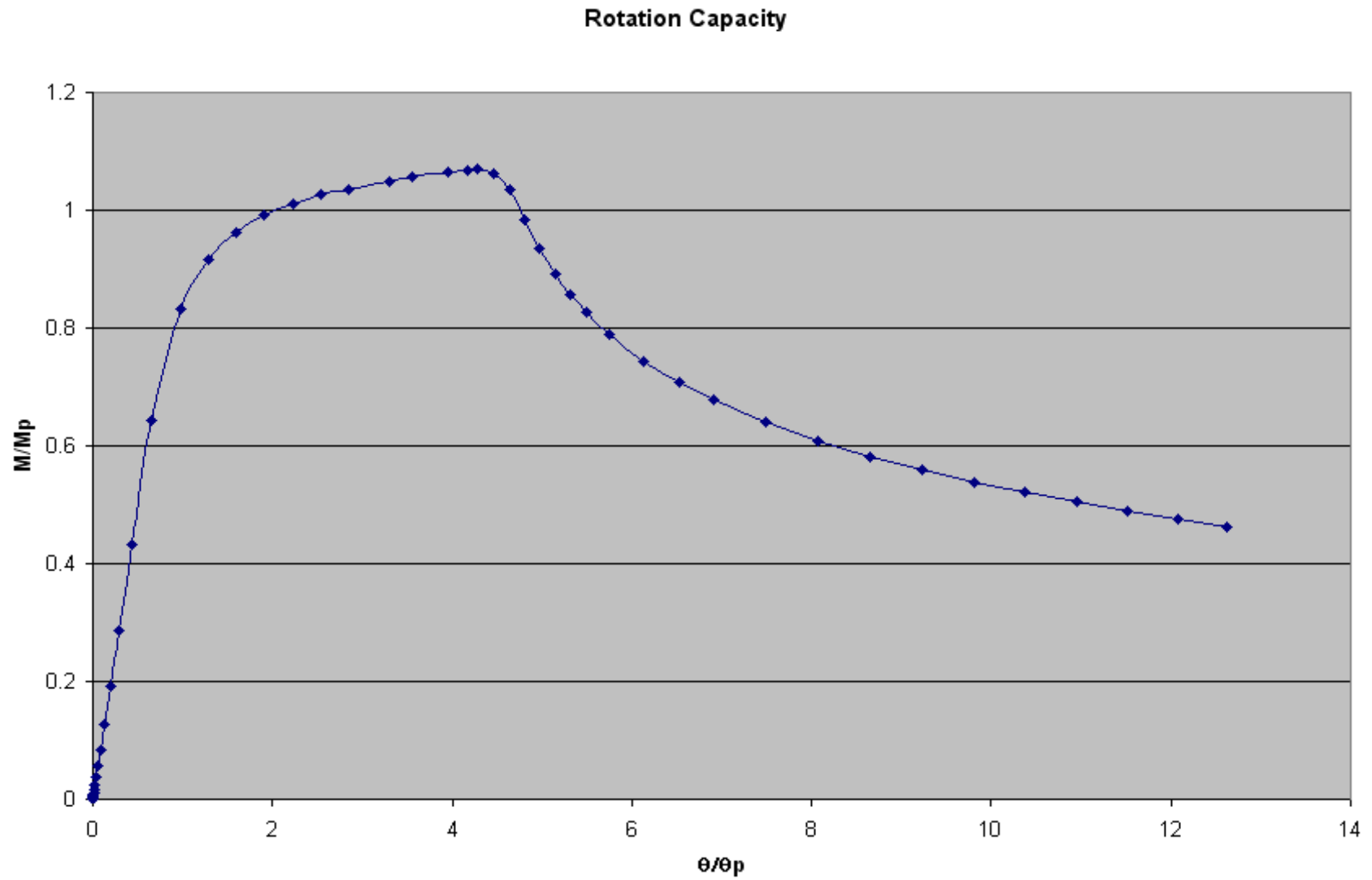


Figure B11 Moment vs. Rotation for WTEE $b_f/d=1.6$, $t_w=1.17''$, $\lambda_p=1.2(65/\sqrt{F_y})$

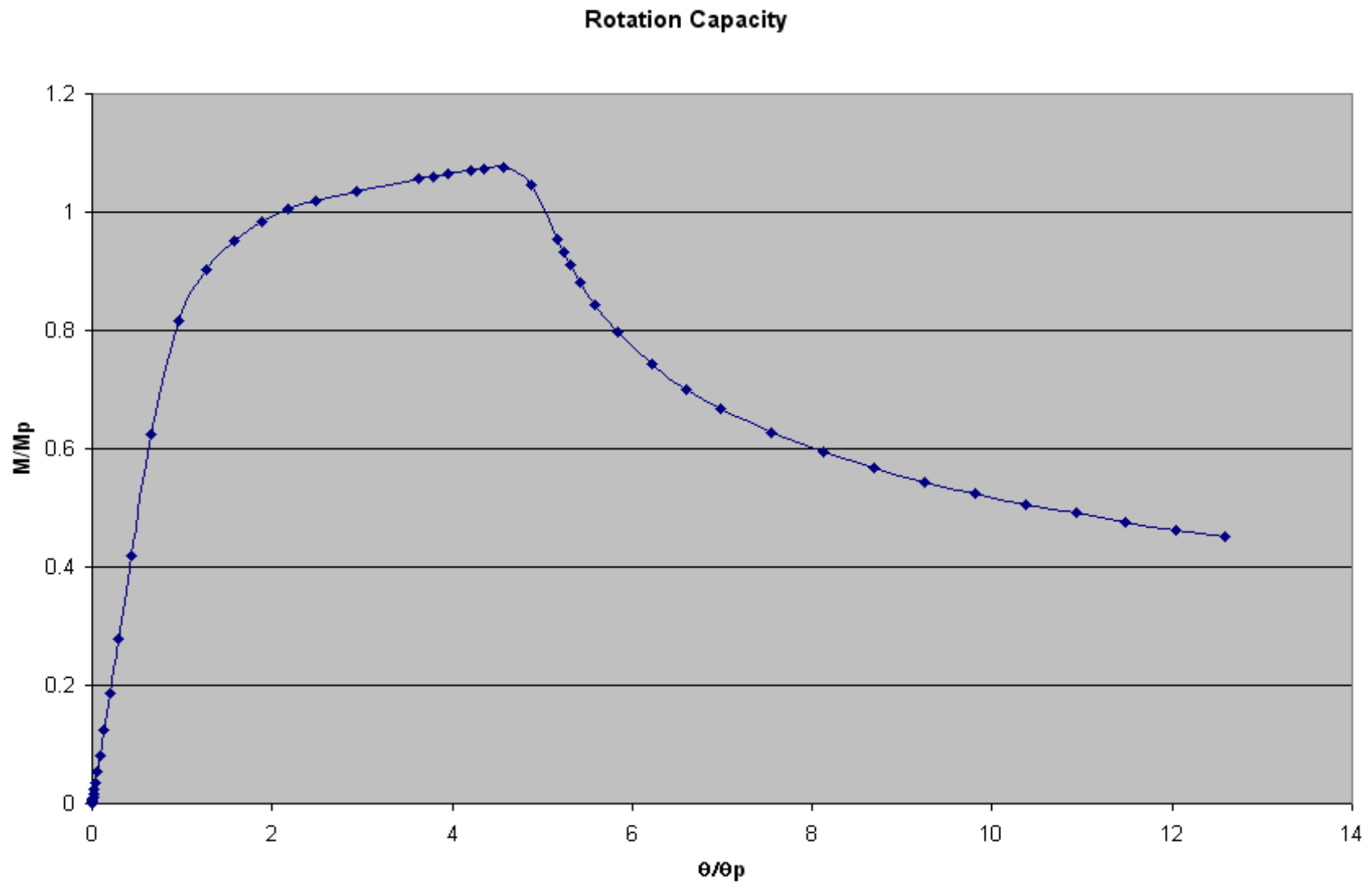


Figure B12 Moment vs. Rotation for WTEE $b_f/d=1.6$, $t_w=1.17''$, $\lambda_p = 1.4(65/\sqrt{F_y})$

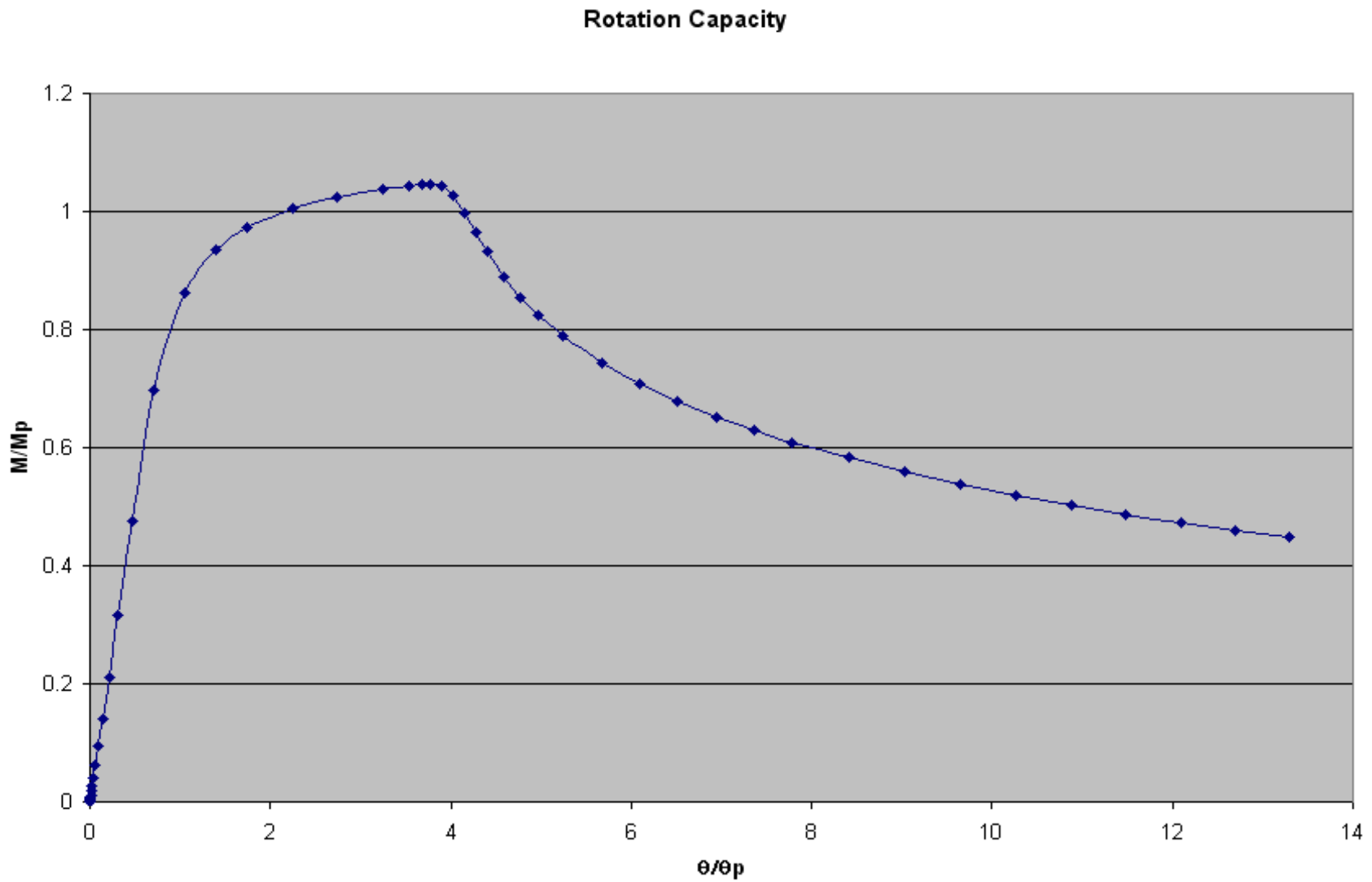


Figure B13 Moment vs. Rotation for WTEE $b_f/d=1.8$, $t_w=1.22''$, $\lambda_p=1.2(65/\sqrt{F_y})$

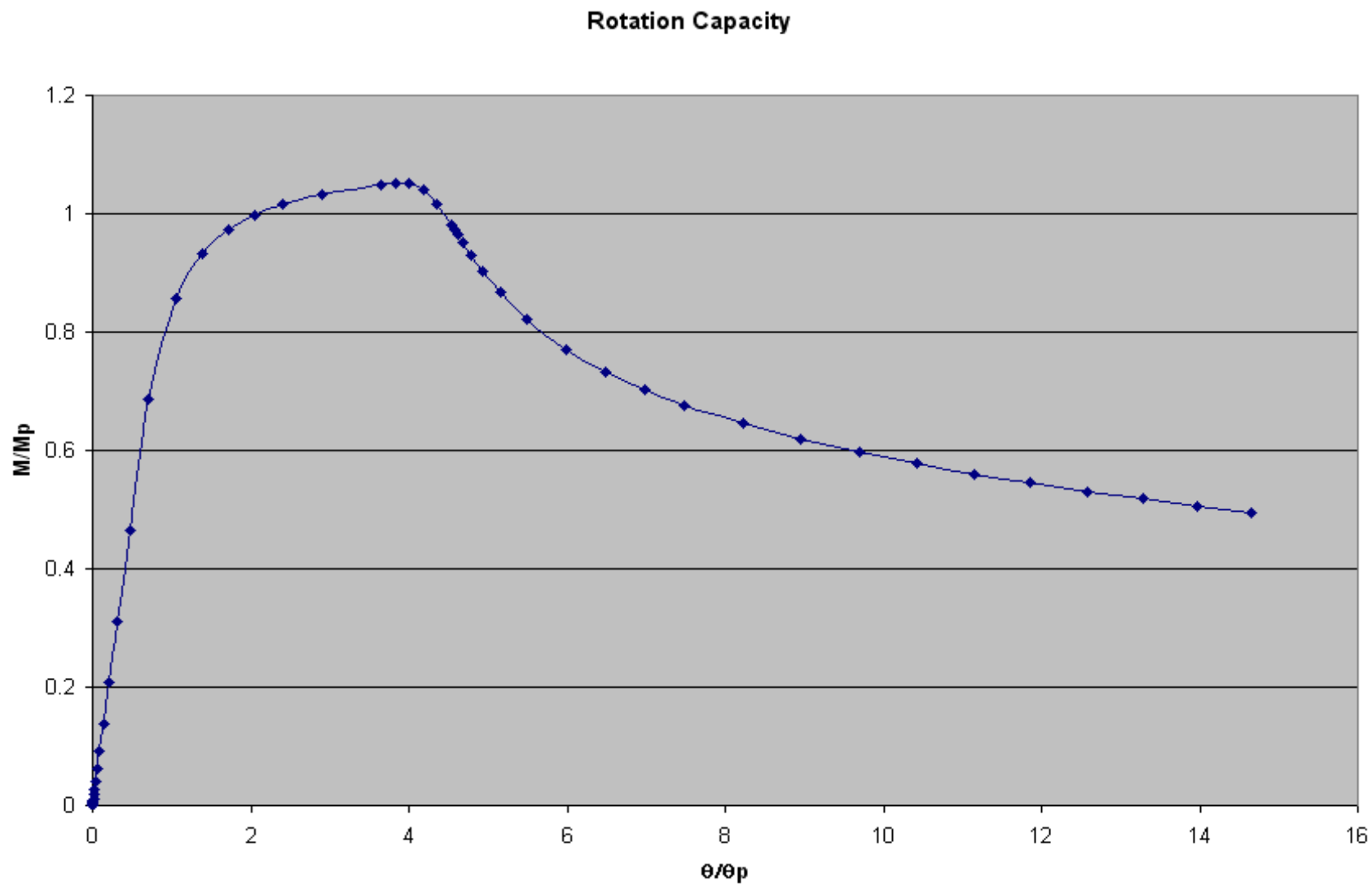


Figure B14 Moment vs. Rotation for WTEE $b_f/d=1.8$, $t_w=1.22''$, $\lambda_p = 1.4(65/\sqrt{F_y})$

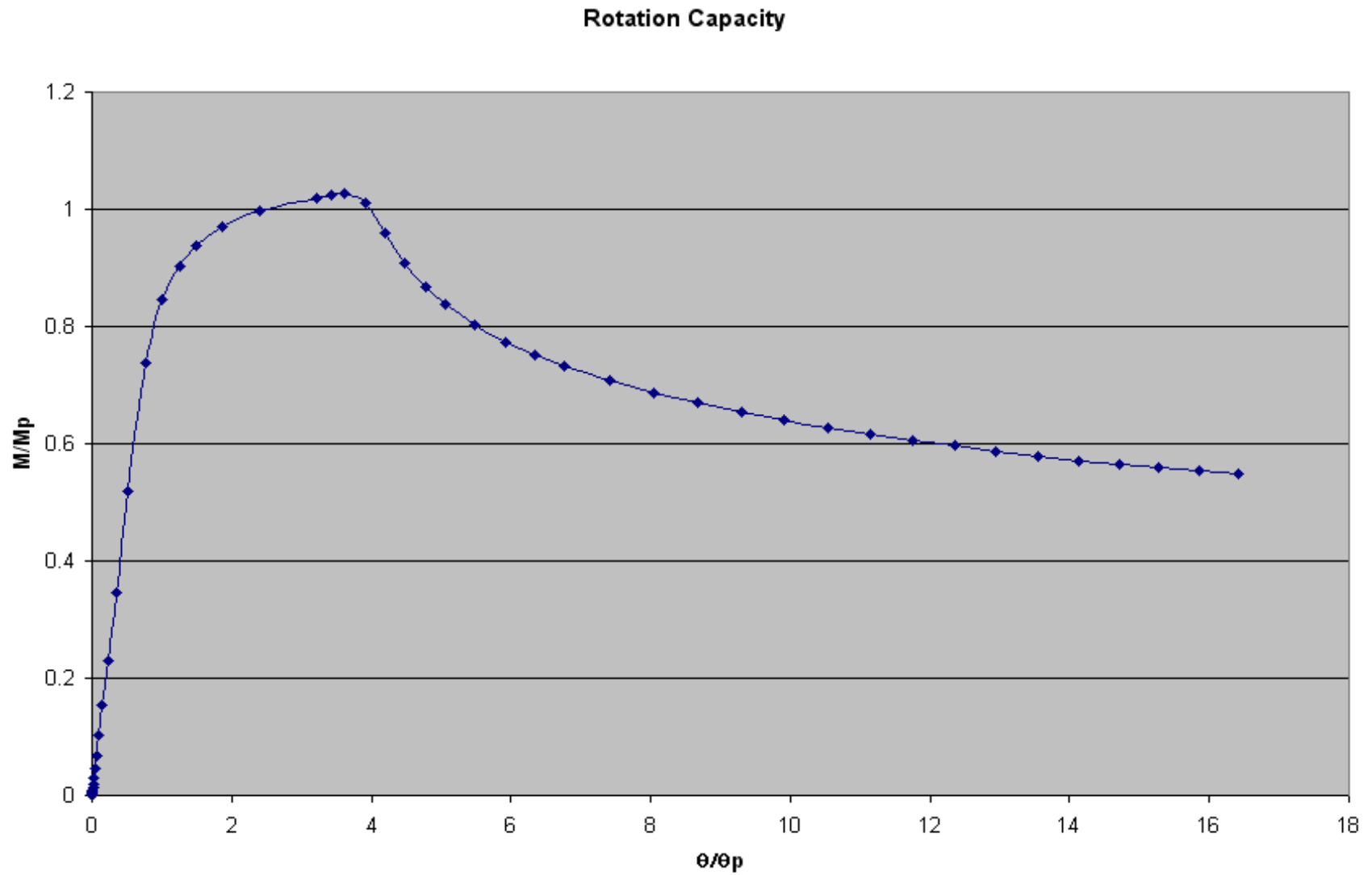


Figure B15 Moment vs. Rotation for WTEE $b_f/d=2.0$, $t_w=1.25''$, $\lambda_p=1.2(65/\sqrt{F_y})$

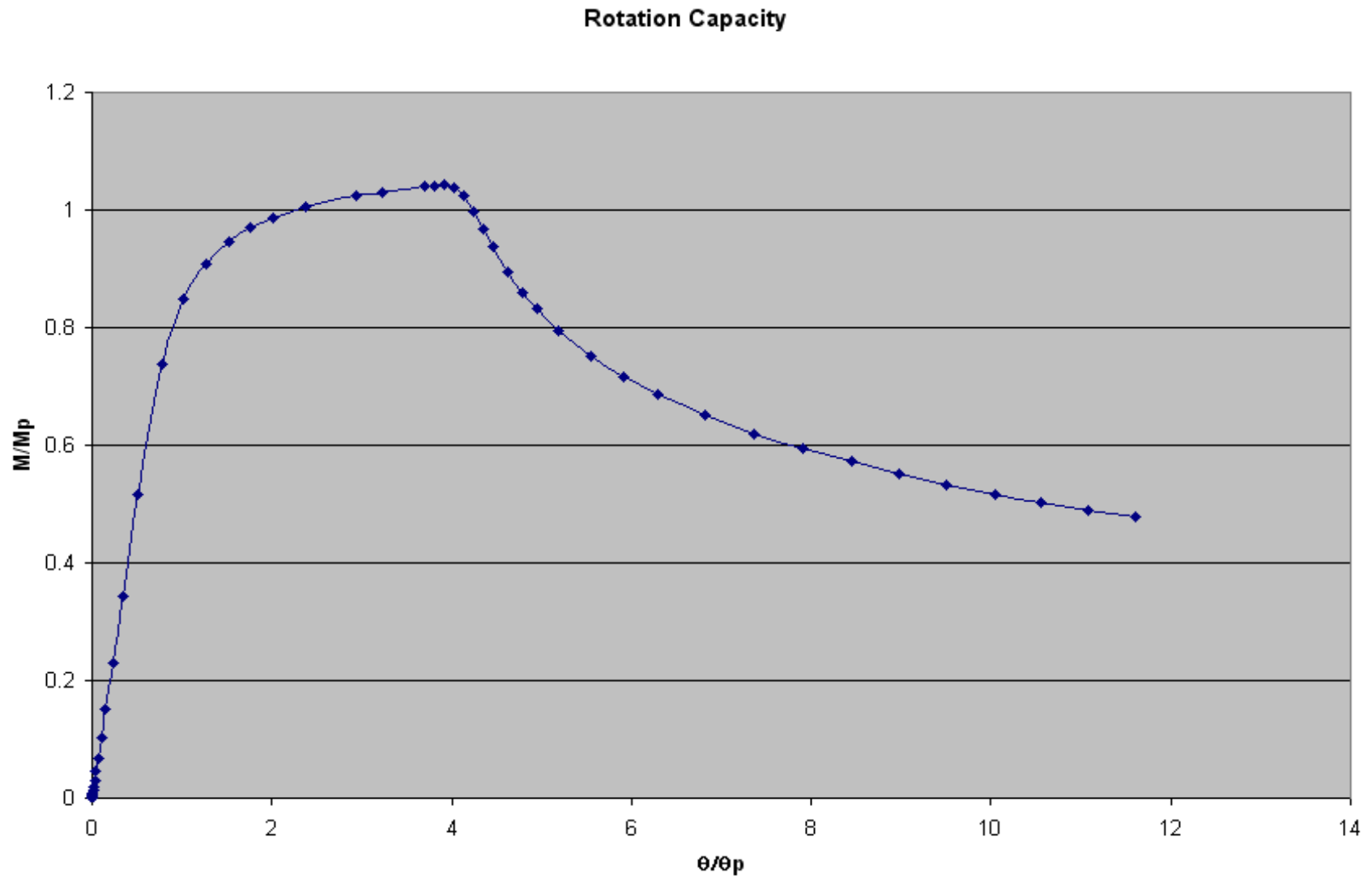


Figure B16 Moment vs. Rotation for WTEE $b_f/d=2.0$, $t_w=1.25''$, $\lambda_p = 1.4(65/\sqrt{F_y})$

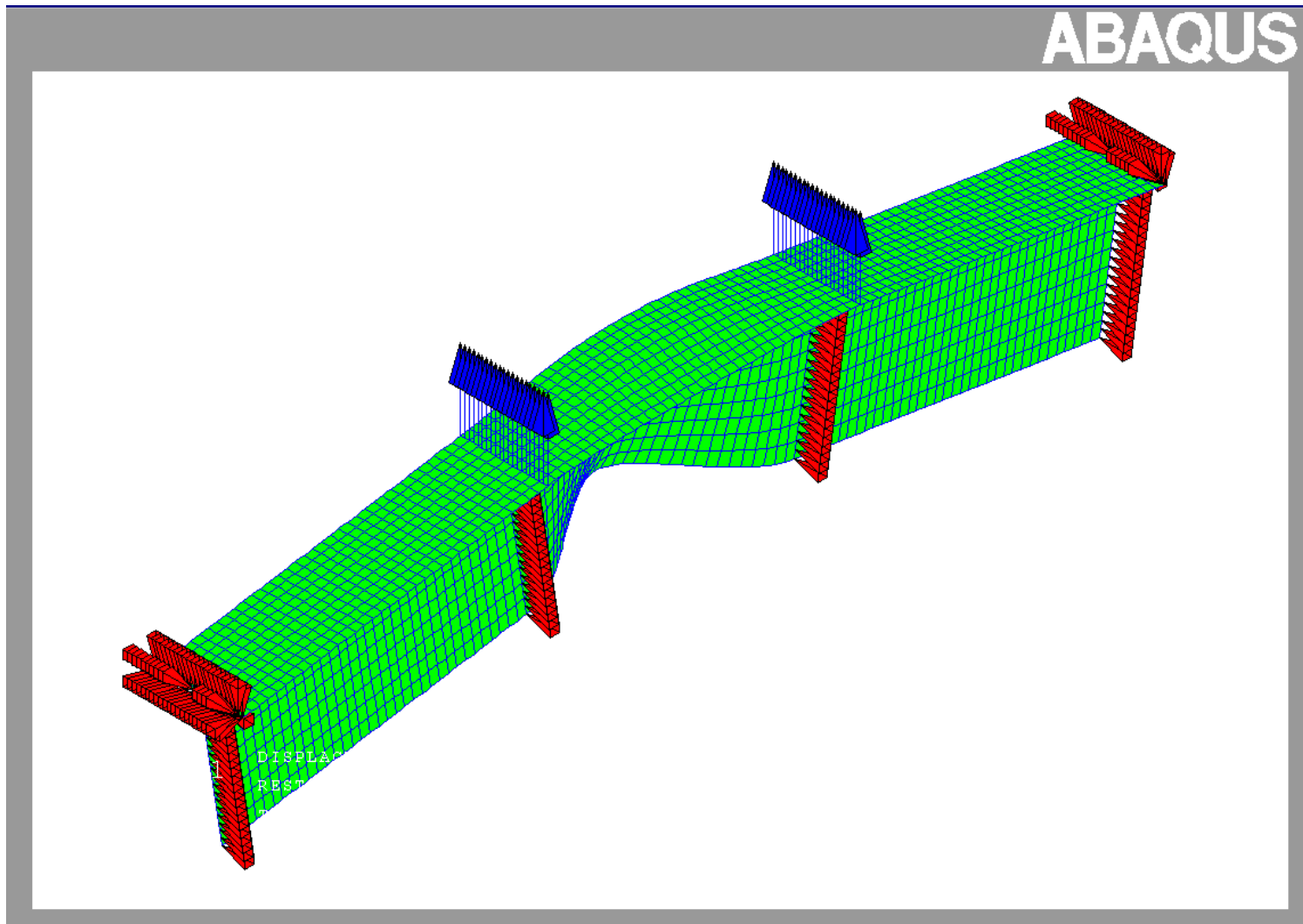


Figure B17 Boundary Conditions and Applied Loading

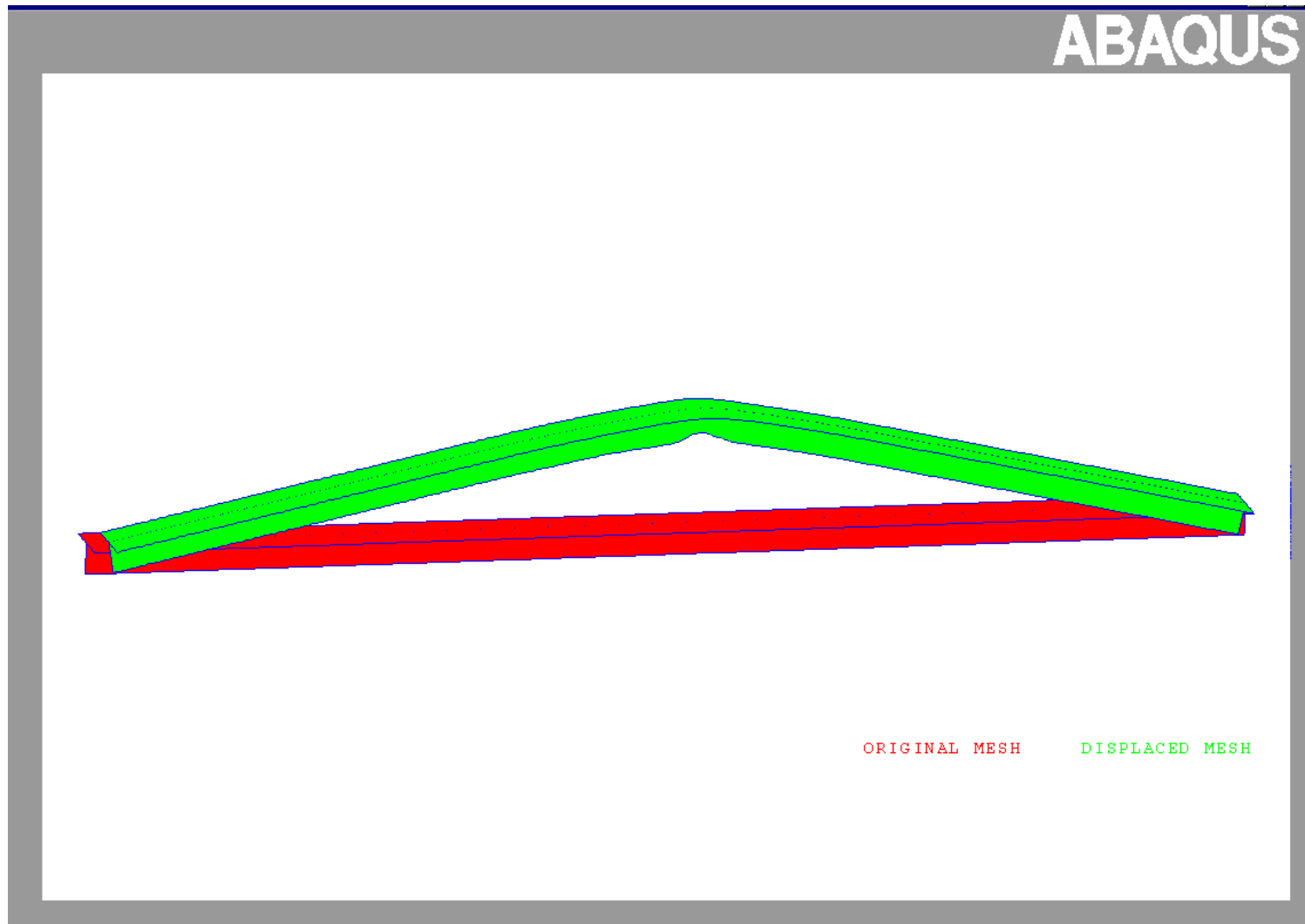


Figure B18 Typical Deformed Shape

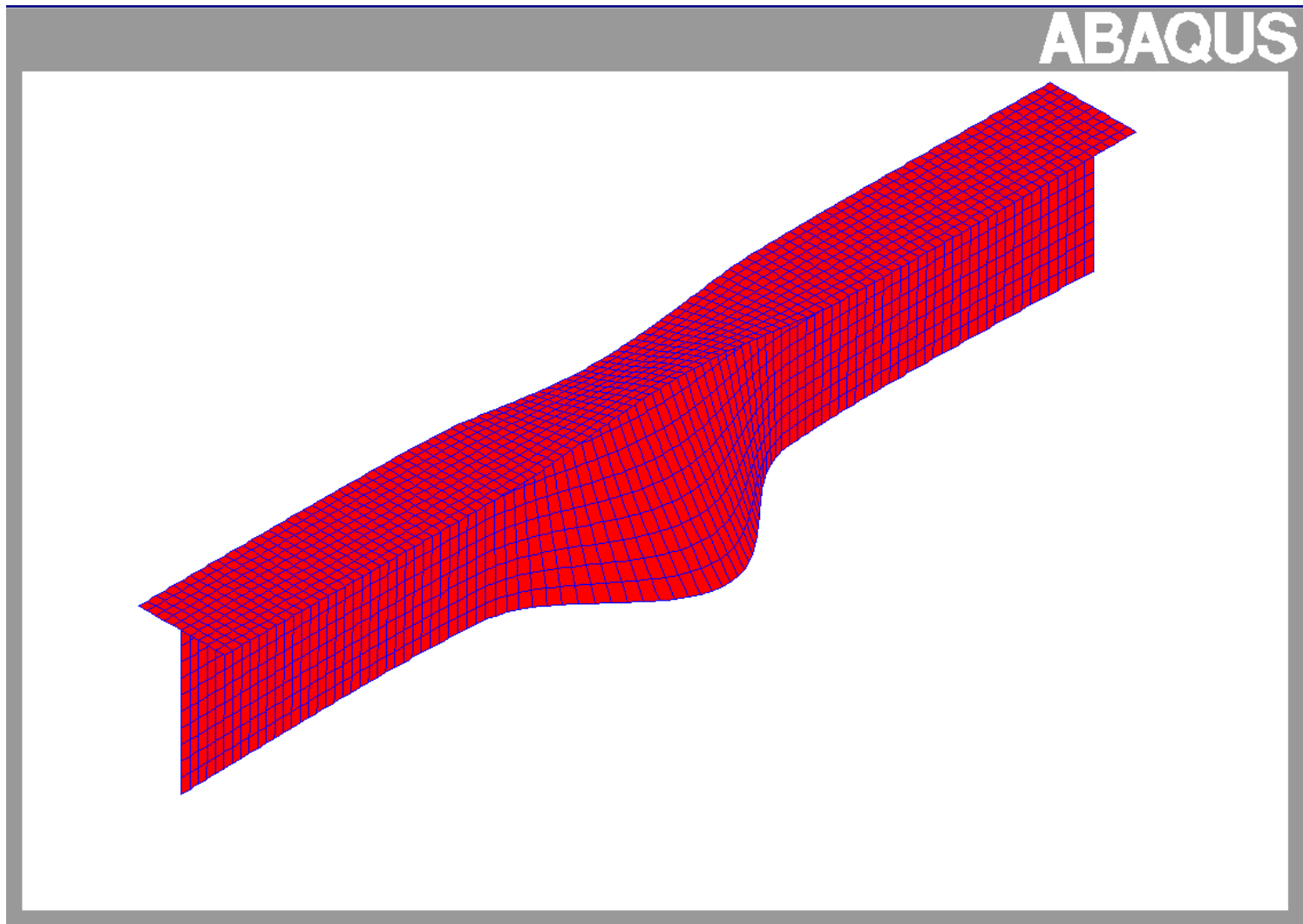


Figure B19 Typical Seed Imperfection

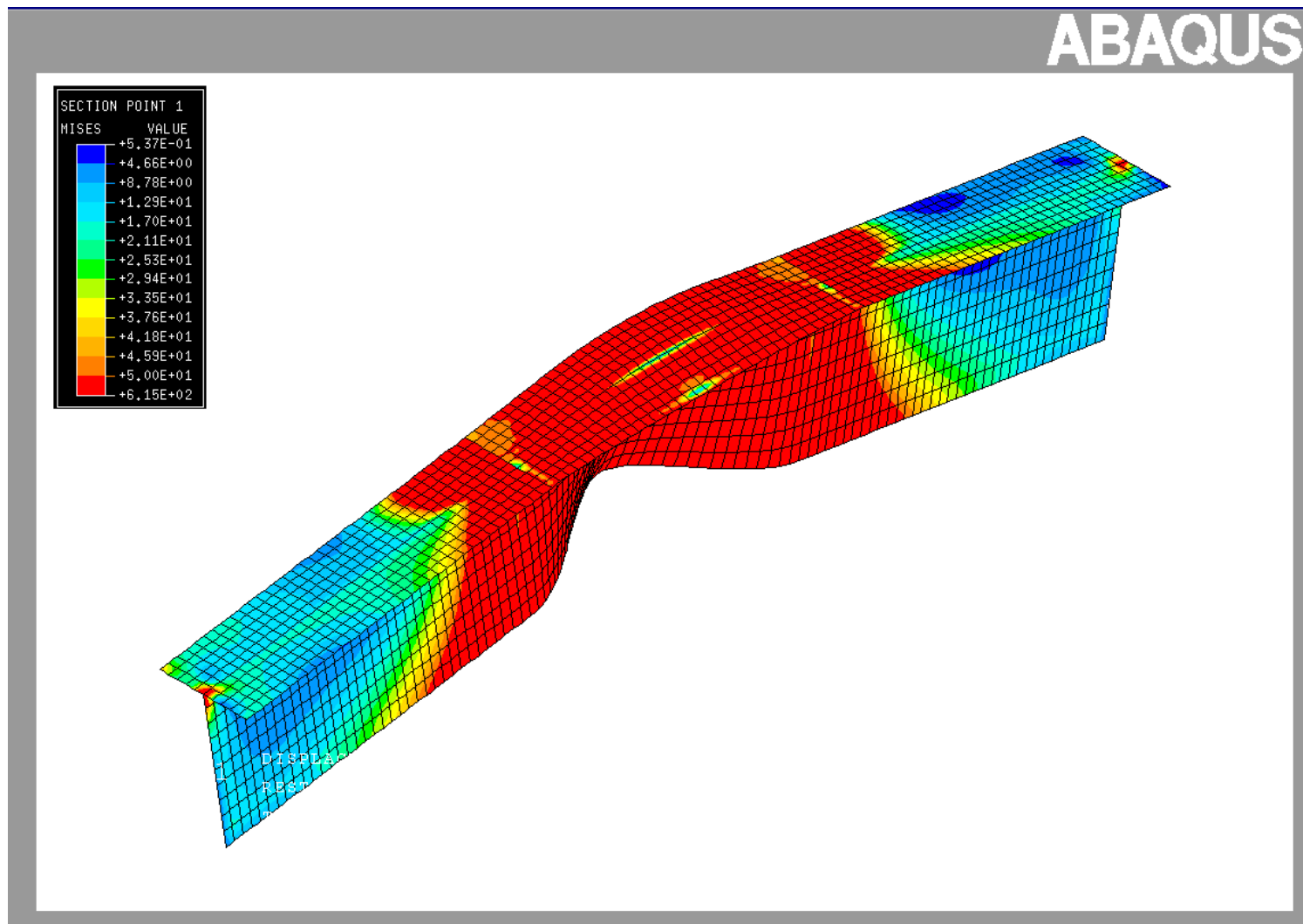


Figure B20 Von Mises Stresses for WTEE $b_f/d=0.6$, $t_w=1.0''$

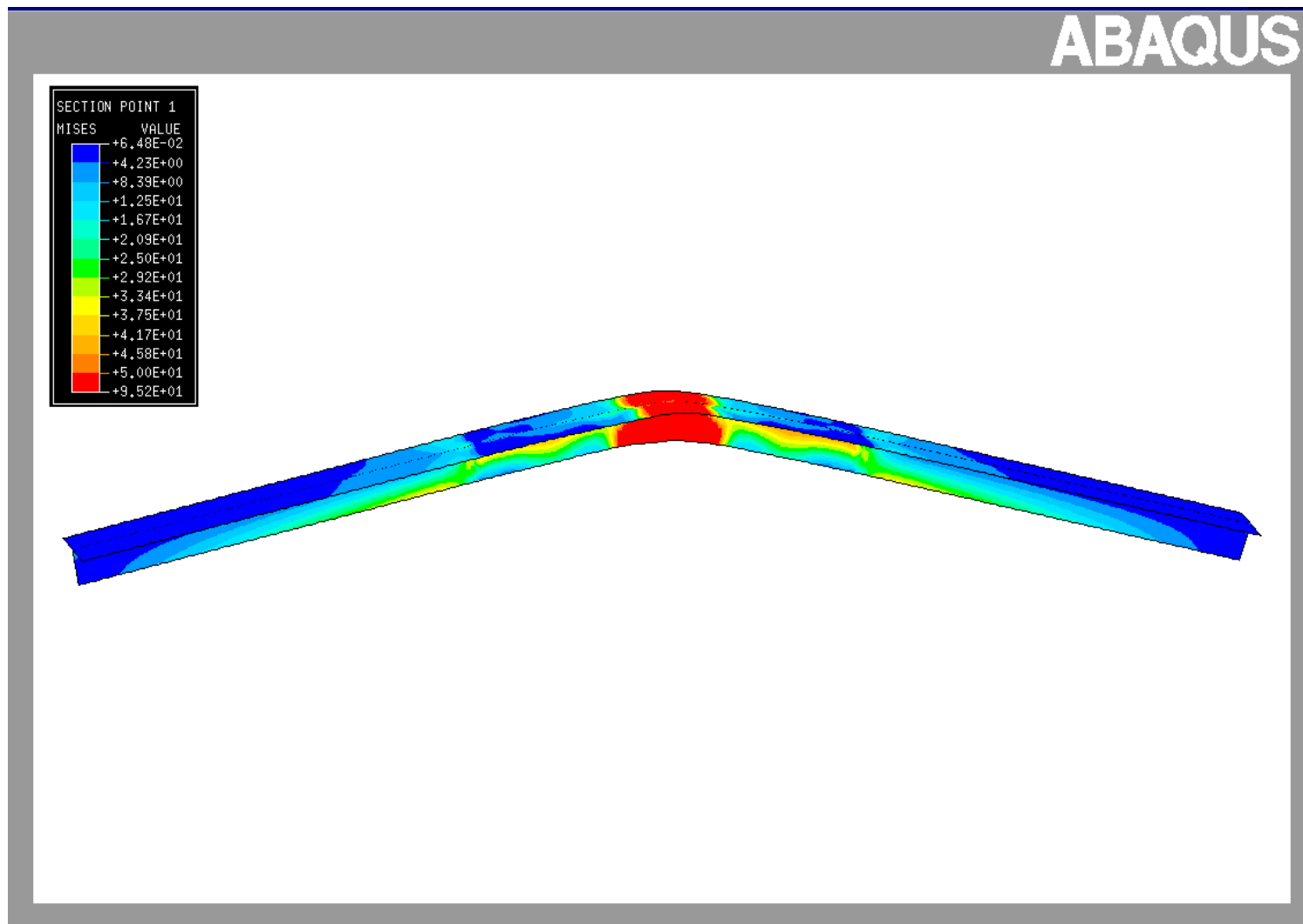


Figure B21 Von Mises Stresses for WTEE $b_f/d=1.8$, $t_w=1.22''$

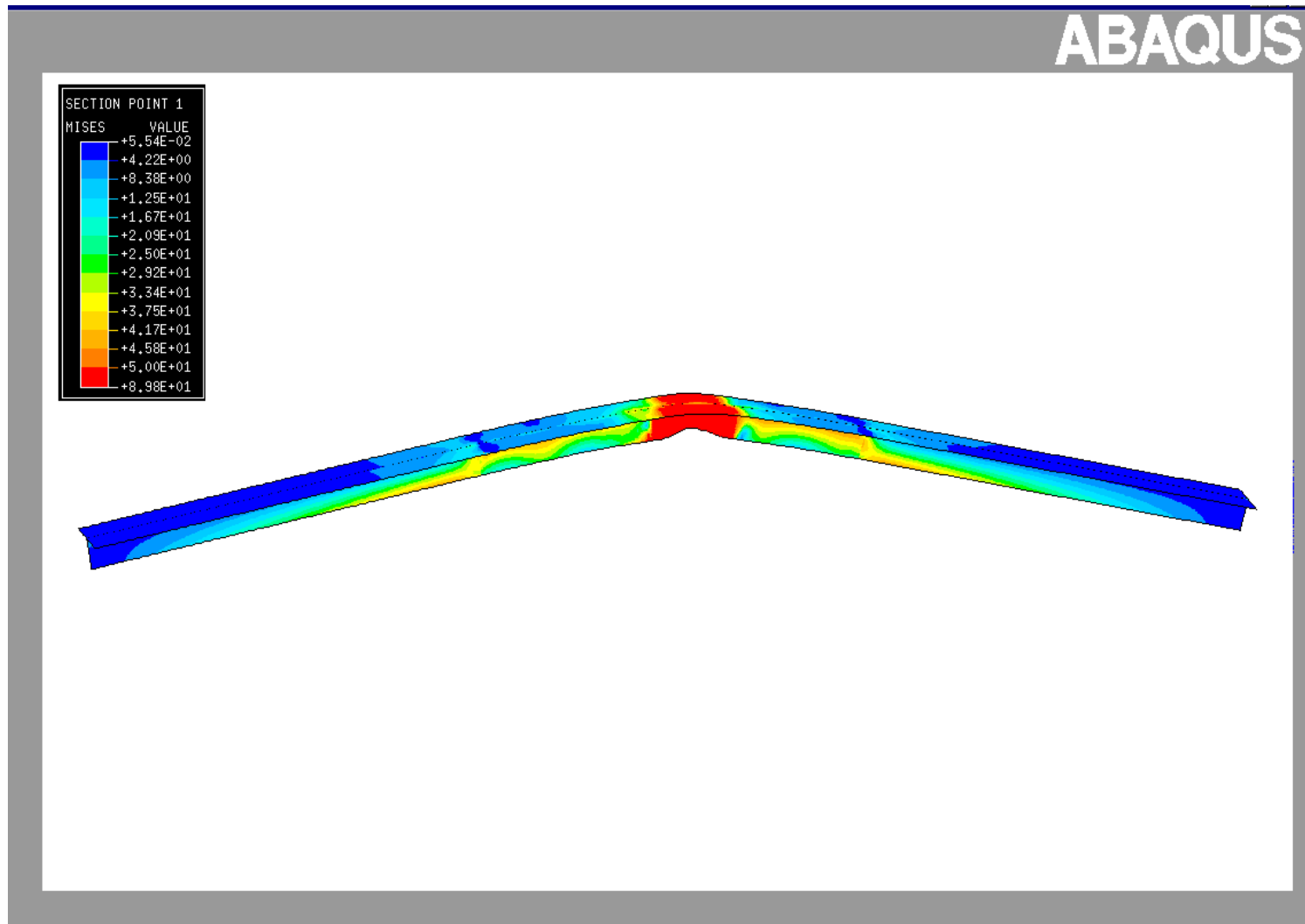


Figure B22 Von Mises Stresses for WTEE $b_f/d=2.0$, $t_w=1.25''$

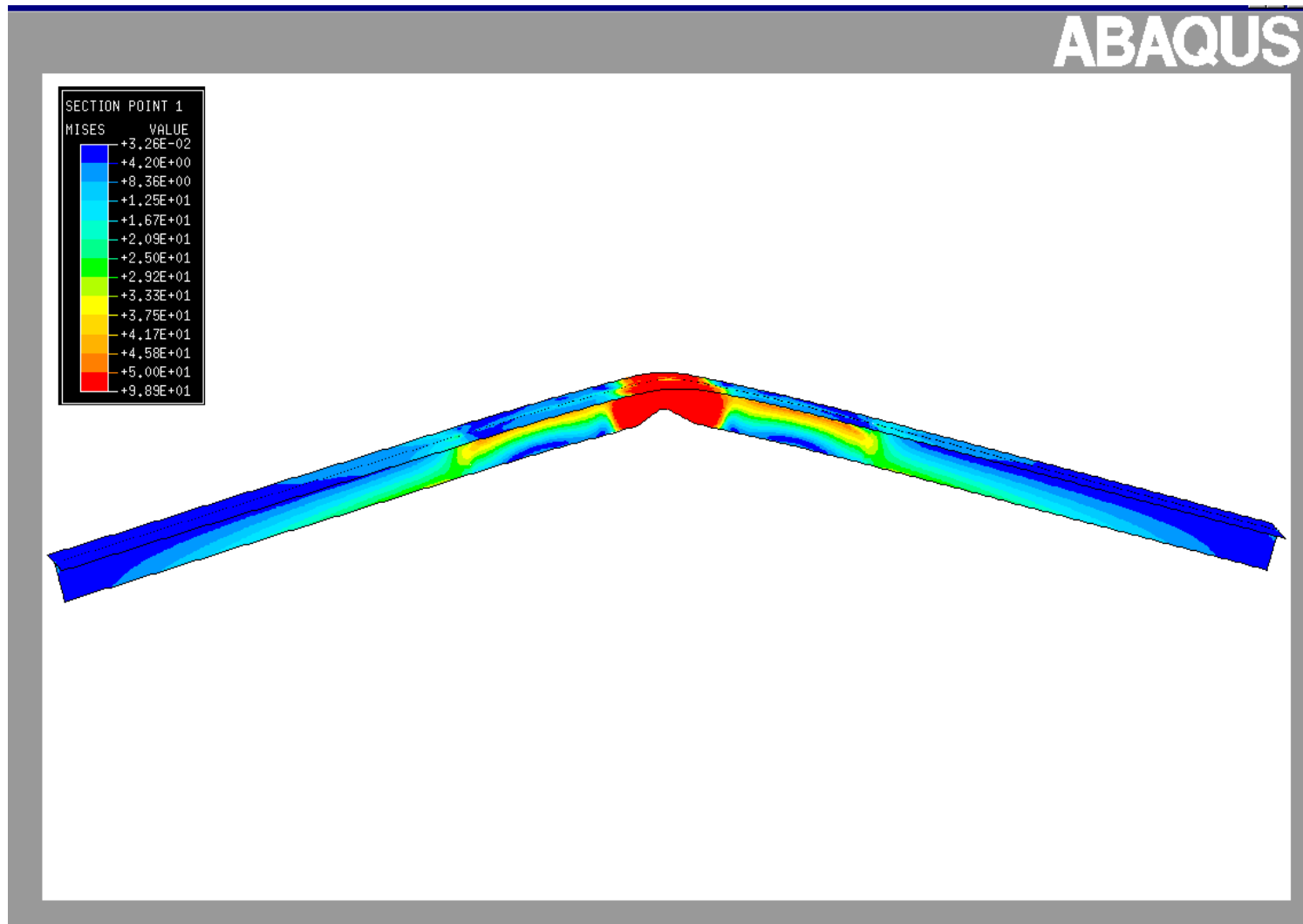


Figure B23 Von Mises Stresses for WTEE $b_f/d=1.8$, $t_w=1.22''$, $\lambda_p = 1.2(65/\sqrt{F_y})$

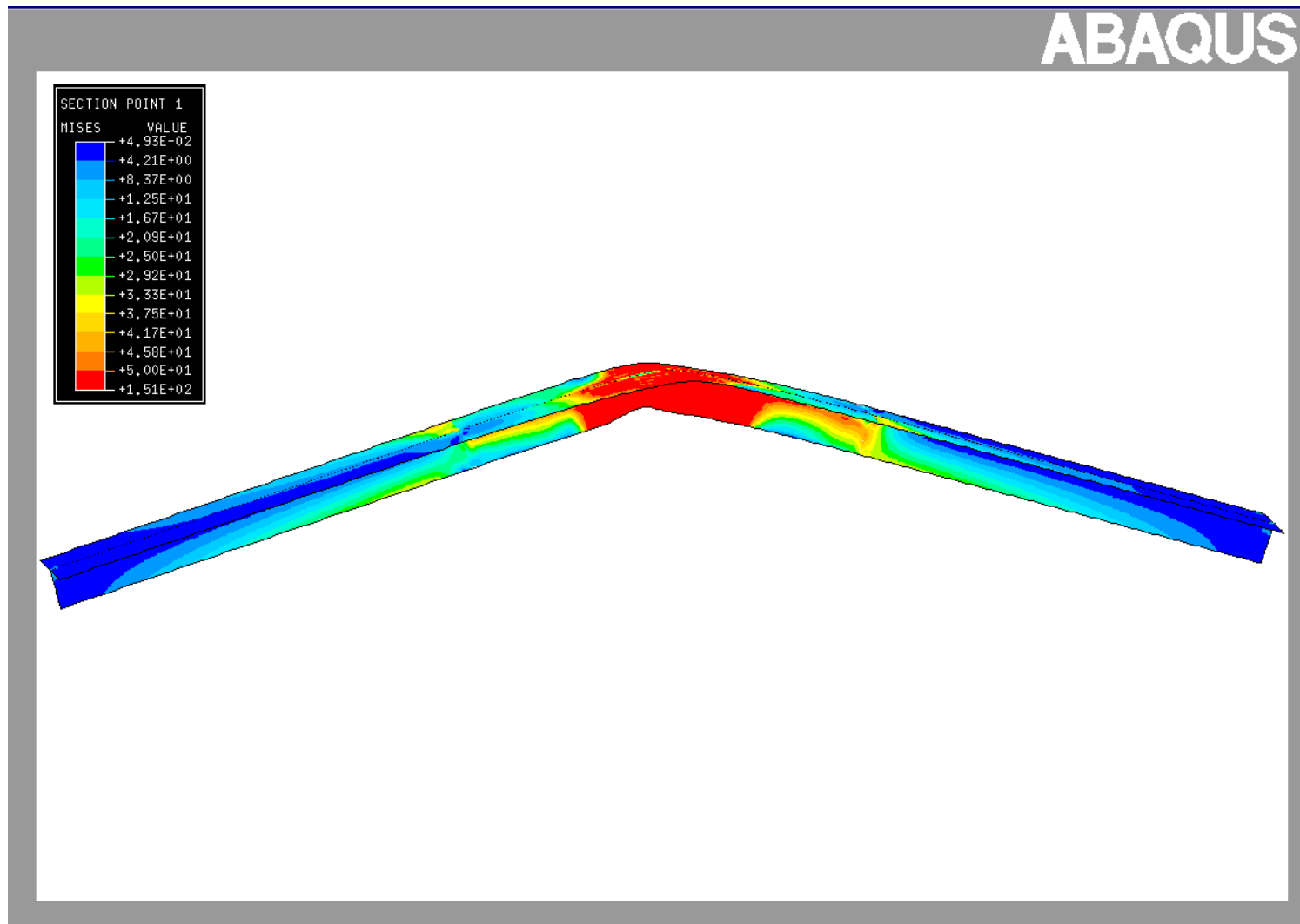


Figure B24 Von Mises Stresses for WTEE $b_f/d=1.8$, $t_w=1.22''$, $\lambda_p = 1.4(65/\sqrt{F_y})$

BIBLIOGRAPHY

1. ABAQUS, (2001), ABAQUS Standard User's Manual, Version 5.8, Volumes 1 to 3, Hibbit, Karlsson & Sorensen, Inc., Pawtucket, Rhode Island, USA.
2. American Institute of Steel Construction (AISC), (1999), *Load and Resistance Factor Design (LRFD) Specification for Structural Steel Buildings*, Chicago, Illinois.
3. American Institute of Steel Construction (AISC), (2001), *Load and Resistance Factor Design (LRFD) Specification for Structural Steel Buildings*, Chicago, Illinois.
4. Beer, F. P., Johnston, Jr., E. R., and DeWolf, J. T., (2002), *Mechanics of Materials*, McGraw-Hill, New York, New York.
5. Chen, W.F., Lui, E.M., (1987), *Structural Stability: Theory Implementation*, Elsevier Science Publishing Co., Inc, New York, New York.
6. Corona E., Ellison, M. S., (1997), "Plastic Buckling of T-Beams Under Pure Bending," *ASCE Journal of Engineering Mechanics*, Vol. 123, May, pp. 466-474.
7. Corona E., Ellison, M. S., (1998), "An Experimental Investigation of T-beam Stability Under Bending," *Experimental Mechanics*, Vol. 38, No. 1, March, pp. 42-47.
8. Ellifritt, D. S., et al., (1992), "Flexural Strength of WT Sections," *AISC Engineering Journal*, Second Quarter, pp. 67-74.
9. Ellison, M. S., Corona, E., (1998), "Plastic Collapse Analysis of T-Beams under Bending," *ASCE Journal of Engineering Mechanics*, August, pp. 818-825.
10. Galambos, T. V., (2001), "Strength of Singly Symmetric I-Shaped Beam-Columns," *Engineering Journal*, Second Quarter, pp. 65-77.
11. Logan, D.L., (1993), *A First Course In the Finite Element Method*, PWS Publishing Company, Boston, MA.
12. Salmon, C. G., Johnson, J.E., (1996), *Steel Structures, Design and Behavior*, Fourth Edition, HarperCollins Publishers Inc., New York, New York.